# *Power and limitations of numerical simulations in General Relativity and other theories of gravity.*

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# Outline

The power is certainly attached to the need to solve Einstein's and matter equations.

The limitations are given by the correct interpretation of the results, or the applicability of the idealized models to real astrophysical situations.

Our code CAFE

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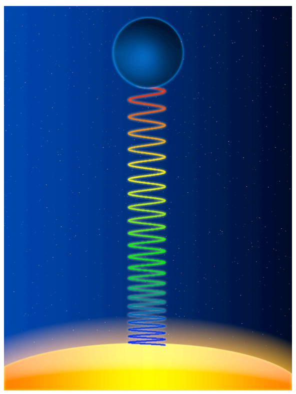
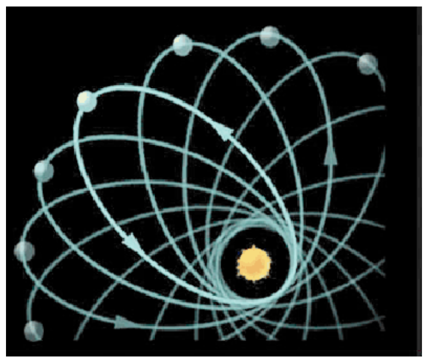
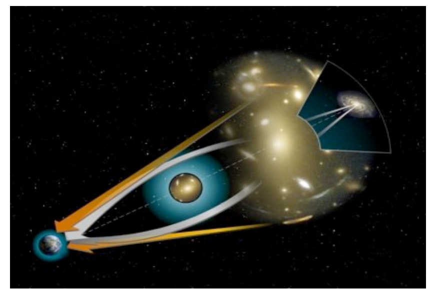
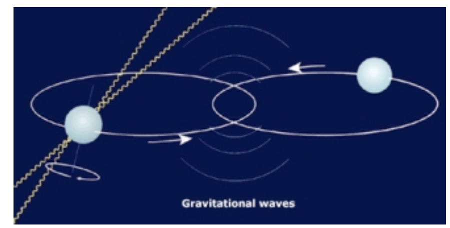
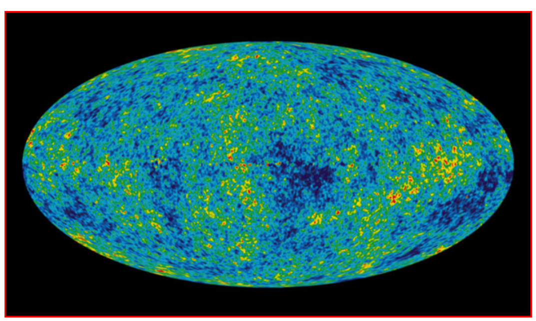
# The power of GR: from theory to astrophysics

GR in the beginning is a nice theory providing interesting geometrical properties of the space-time….

Until tests in the weak field limit were available


Or started getting into controversy at cosmic scale.



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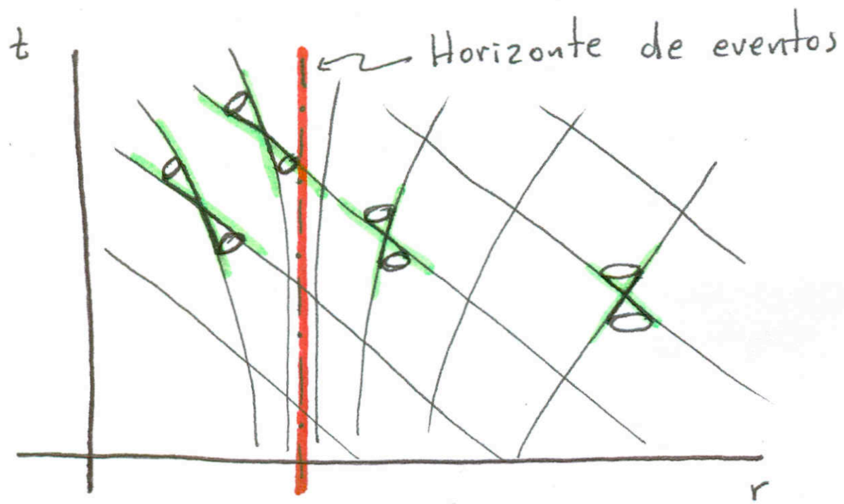
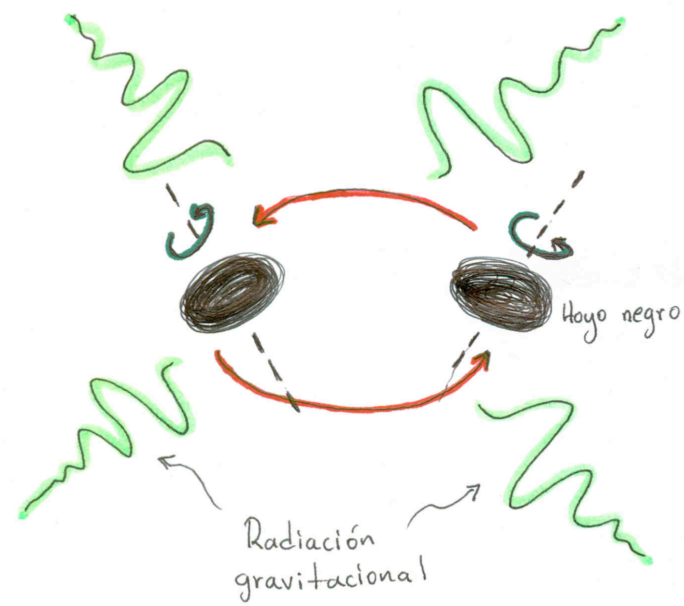
# Until the astrophysics of GR came along

Existence of black holes

Existence of gravitational waves

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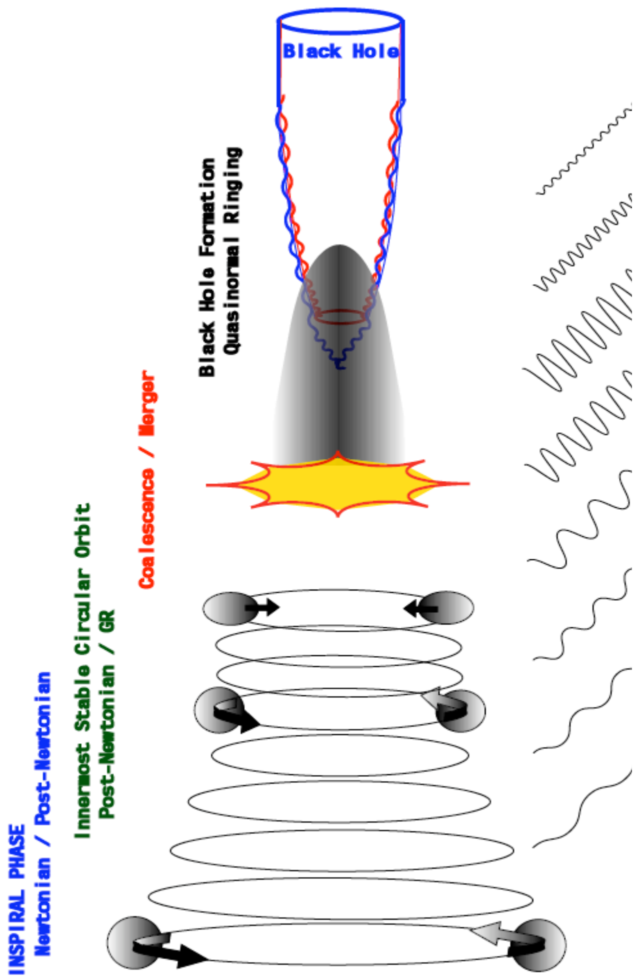
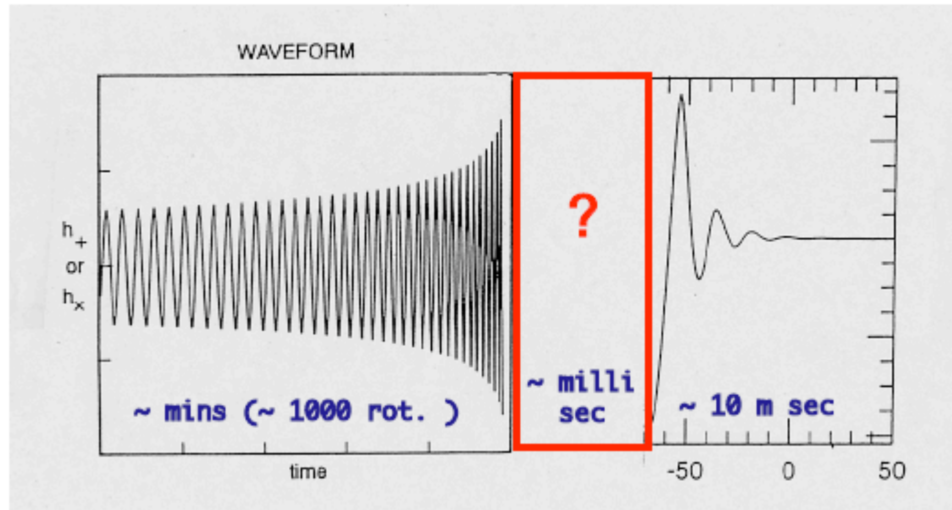
# Global network of detectors


- Simultaneous detection (within msecs)
- Detection confidence
- Sky location
- Source polarization
- Verify light speed propagation

# The binary black hole problem was THE problem




INSPIRAL → COALESCE → BLACKHOLE FORMATION

Innermost Stable Circular Orbit?

Post Newtonian Approx.

→ Numerical Relativity

→ BH. Perturbation

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$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \frac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\frac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}K^2\right)$$

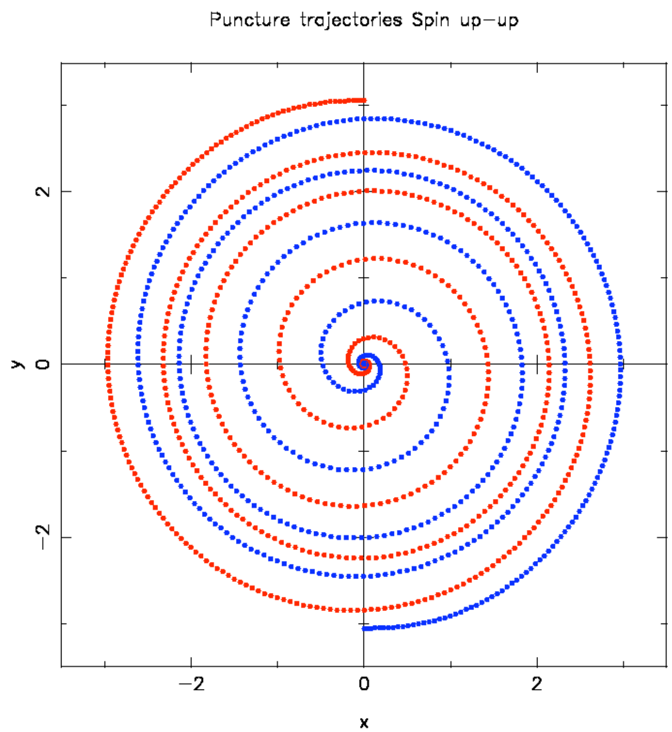
$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \frac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$

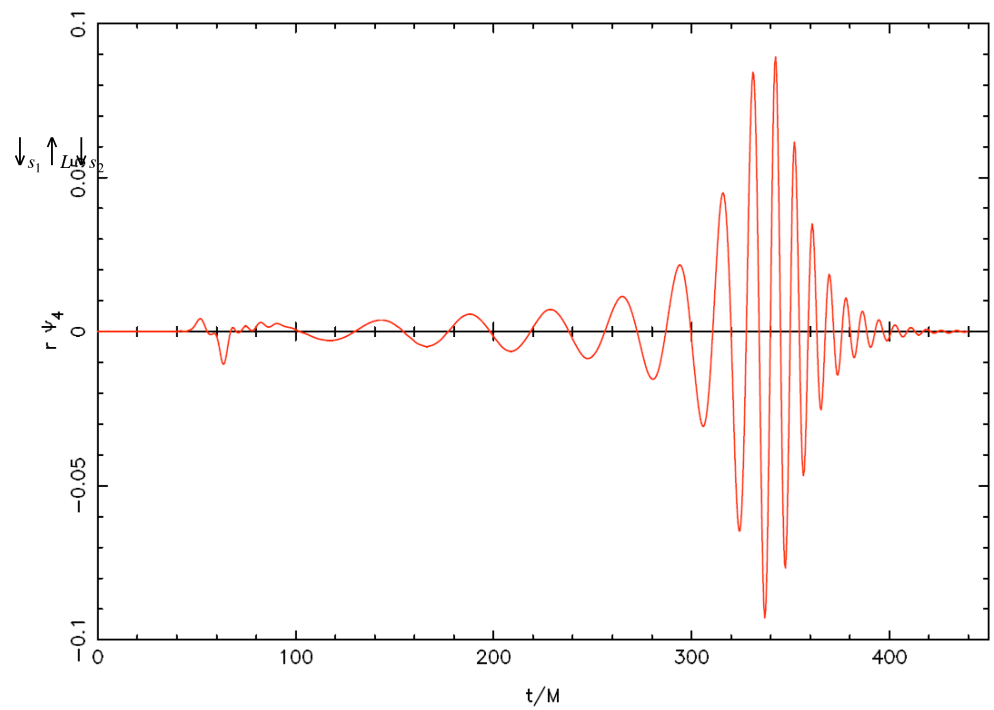
$$+\frac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \frac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \frac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

# With various spin configurations

$\uparrow s_1 \uparrow L \uparrow s_2$

Convergence was usual at this very time (~2006-2007)

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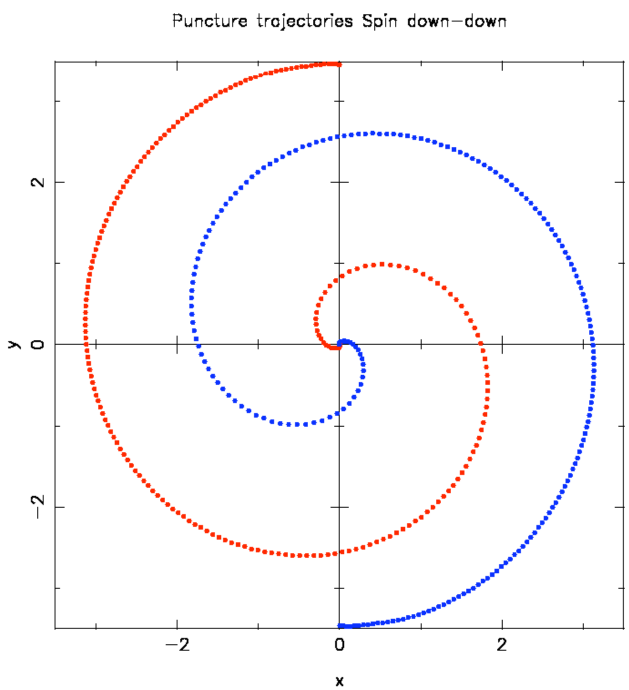
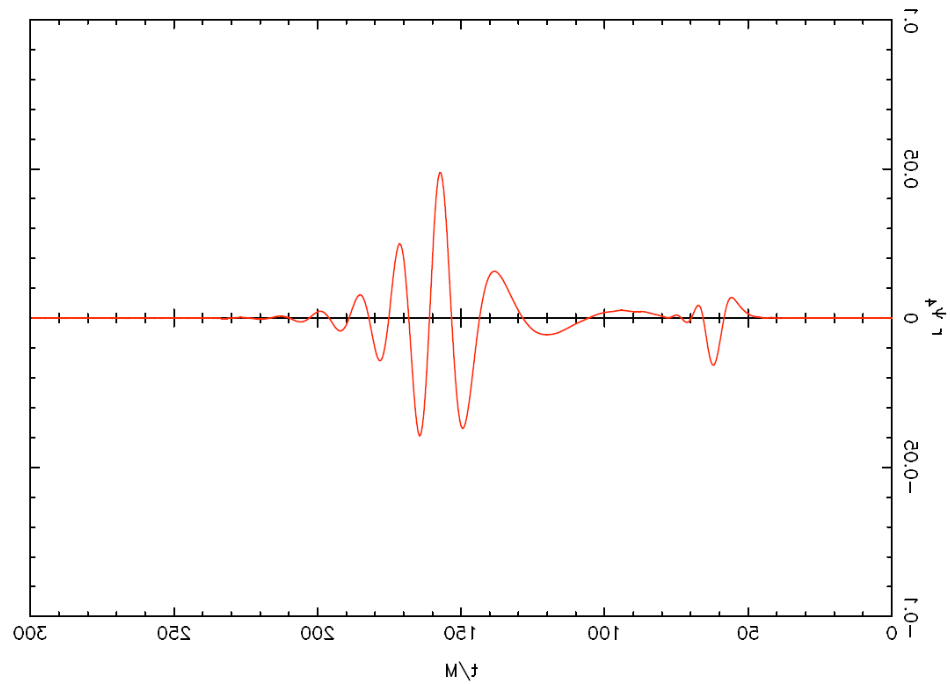
# In all these results there was $\downarrow s_1 \uparrow L \downarrow s_2$ something called CONVERGENCE



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# The astrophysical model

- Like in the rest of astronomical observations, it is necessary to count with models that allow filtering out from “noise”.
- In a binary black hole collision there are many parameters: spin1, spin 2, mass 1, mass 2, L, position of the source, etc.
- <u>**The inverse problem is impressive.**</u>
- Each possible signal requires the solution of Einstein’s equations, a system of PDEs solvable only with a computer (the power again).
- The problem is now computational: the construction of possible wave signals and filtering signals diving in a sea of big data.
- The problem is that what is called an observed black hole is actually a bunch of signals in the EM spectrum: there is MATTER, that is the matter.

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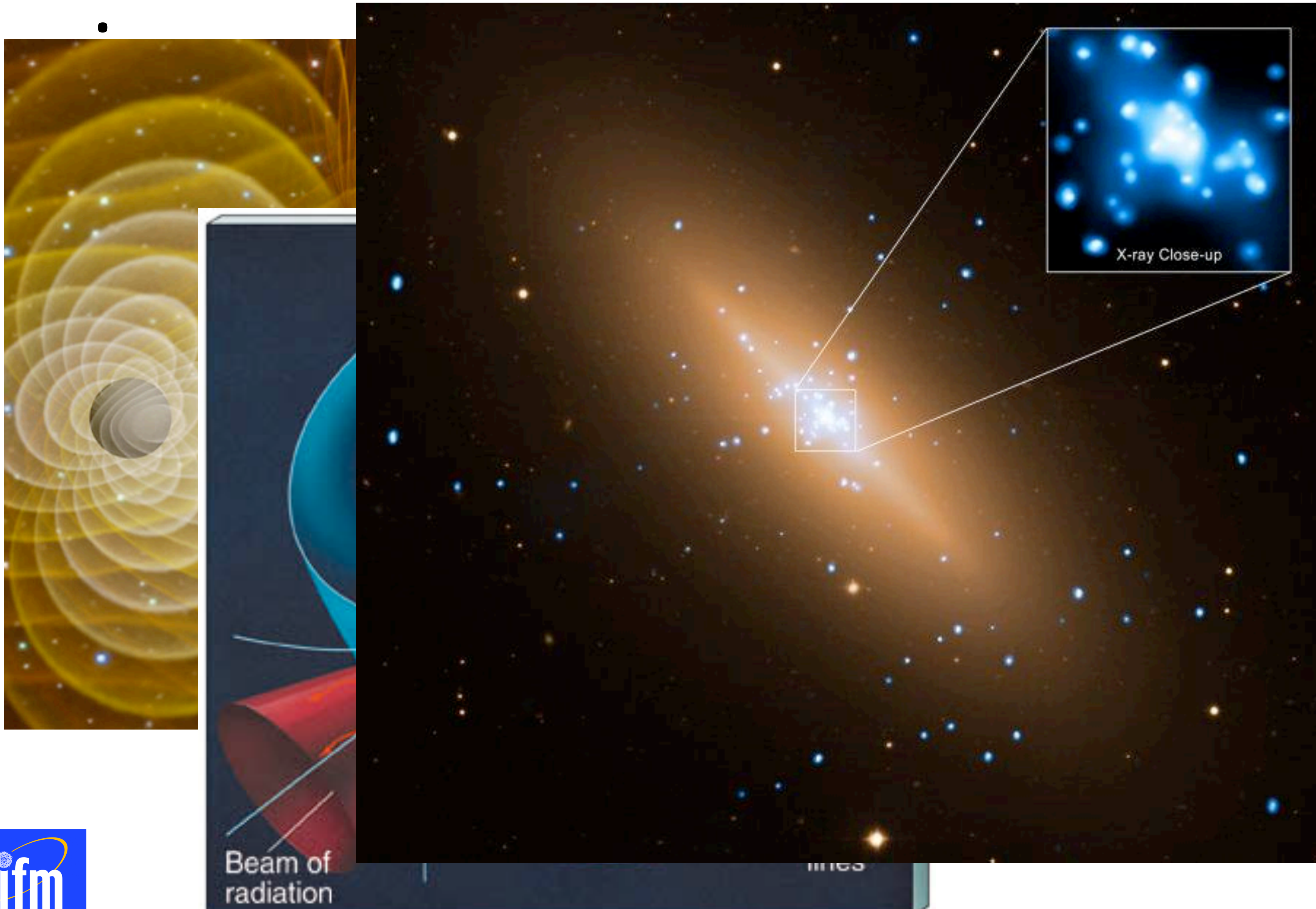
# Binary black hole problem and similar other problems and their astronomy….



# The system of equations and the numerical implementation required in astrophysical systems

In general these systems are submitted to STRONG GRAVITATIONAL FIELDS => ***Einstein's equations***

The gas involved in the processes obeys a given model => ***Relativistic Euler equations***

Magnetic fields play an important role => ***Maxwell's equations***

Radiative processes => Some flux transport model for radiation or similar fields

THEN the system of equations requires a robust numerical code capable of solving *combinations* of these sets of equations

JETS
CORE COLLAPSE
BLACK HOLES
ACCRETION DISKS
BINARY NEUTRON STARS

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# Nowadays relativistic astrophysics

Cactus Einstein Toolkit, a multi usage package capable of solving the general relativistic MHD.

Whisky, a code that in its more sophisticated version can evolve general relativistic resistive magnetohydrodynamics \citep{whisky}, based on Cactus.

GENESIS, which is a code capable of solving the GRMHD equations for relativistic flows and stellar core collapse in general relativity [DeBrye, Cerda, Aloy, Font.]

HARM, a general relativistic code for a fixed space-time and its latest version including radiation terms [Gammie, McKinney, Toth].

HAD, that in its most recent version is capable of evolving binary compact stars in the presence of magnetic fields, in general relativity.

There are also independent codes, e.g. the onecapable of dealing with general relativistic hydrodynamics developed by Pretorius .

CoCoNuT, evolves the General relativistic magneto-hydrodynamics to simulate core collapse of massive stars and the evolution of newtron stars [Cerda-Duran et al.].

Specific purpose codes also include [Penner], designed to evolve the accretion of magnetized winds onto black holes. The PLUTO code, capable of solving RMHD equations with AMR [Mignone, Colella, et al.]

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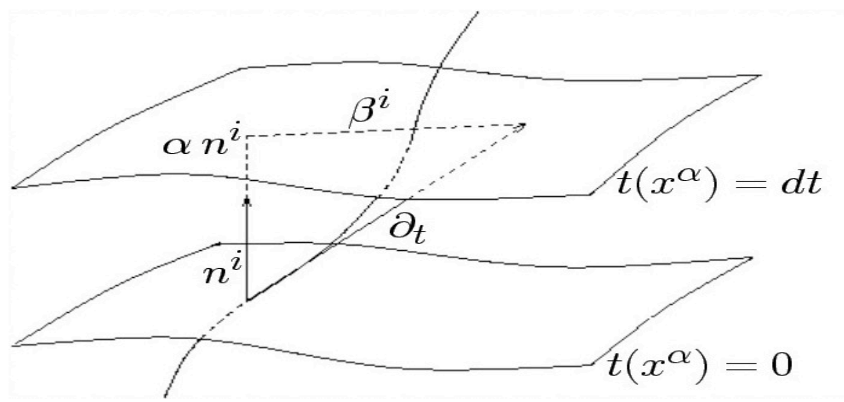
# The fluid model

$$T^{\mu\nu} = \rho_0 h u^\mu u^\nu + p g^{\mu\nu}$$

$$p = (\Gamma - 1)\rho\varepsilon$$

$$\nabla_\nu(\rho_0 u^\nu) = 0$$

$$\nabla_\nu(T^{\mu\nu}) = 0$$

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# RHD equations

$$\frac{\partial u}{\partial t} + \frac{\partial F^i(u)}{\partial x^i} = S$$

$$u = \begin{bmatrix} D \\ J_1 \\ J_2 \\ J_3 \\ \tau \end{bmatrix}, F^i = \begin{bmatrix} \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)D \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_1 + \alpha\sqrt{\gamma}\,p\,\delta^i{}_1 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_2 + \alpha\sqrt{\gamma}\,p\,\delta^i{}_2 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_3 + \alpha\sqrt{\gamma}\,p\,\delta^i{}_3 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right) + \alpha\sqrt{\gamma}\,v^i p \end{bmatrix}, S = \begin{bmatrix} 0 \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^{\sigma}{}_{\mu 1} \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^{\sigma}{}_{\mu 2} \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^{\sigma}{}_{\mu 3} \\ \alpha\sqrt{\gamma}(T^{\mu 0}\partial_{\mu}\alpha - \alpha T^{\mu\nu}\Gamma^{0}{}_{\mu\nu}) \end{bmatrix}$$

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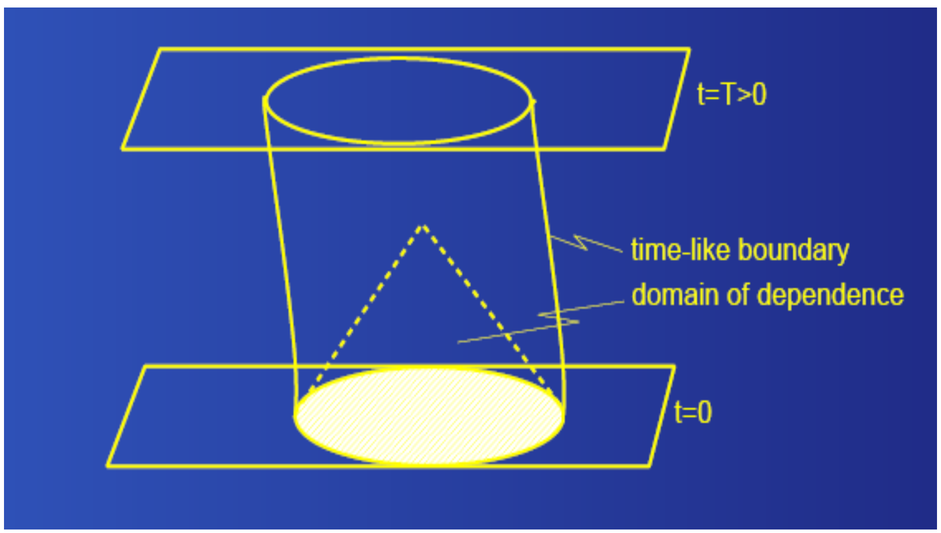
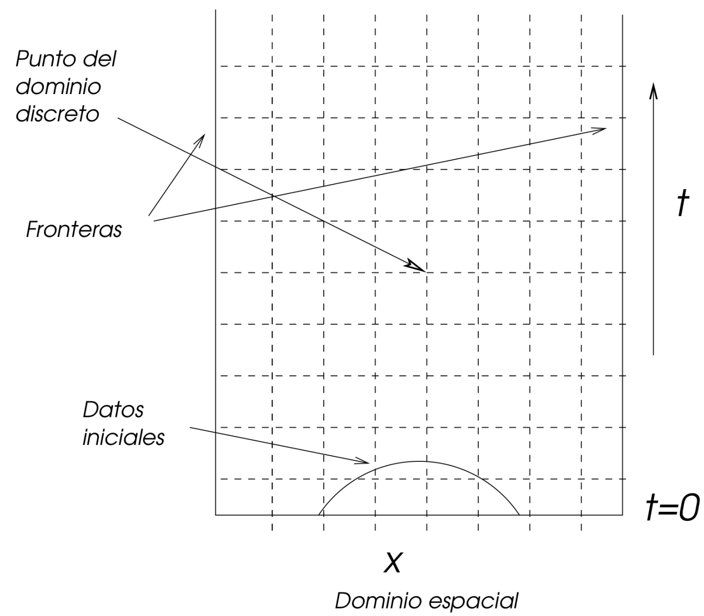
# Basic RHD & RMHD

- These equations develop discontinuities of smooth initial data

- This is the reason to test any application with discontinuous solutions

- This is also the reason to use finite volume methods

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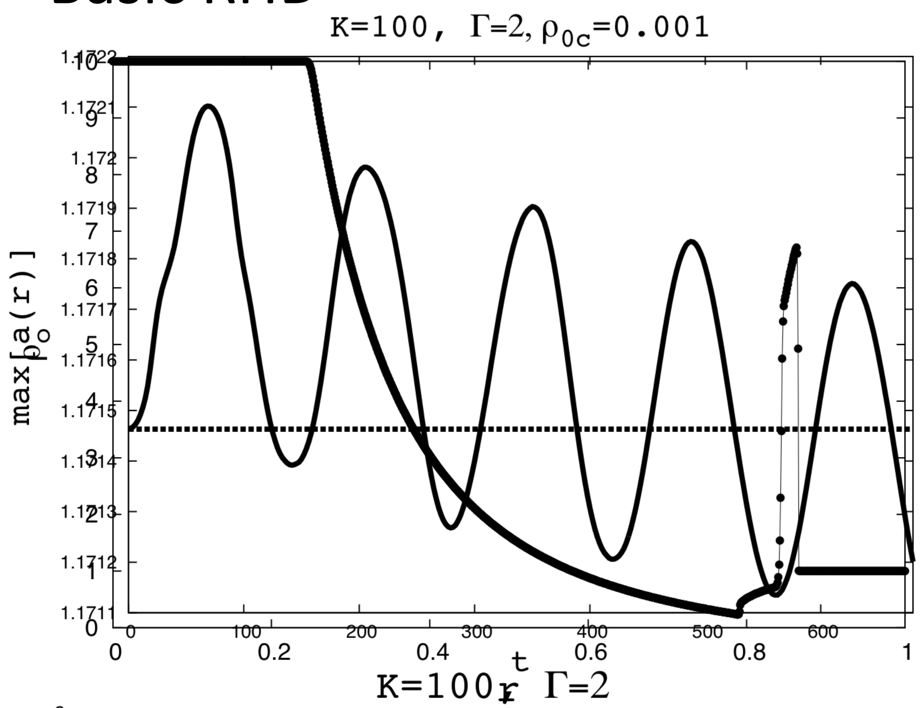
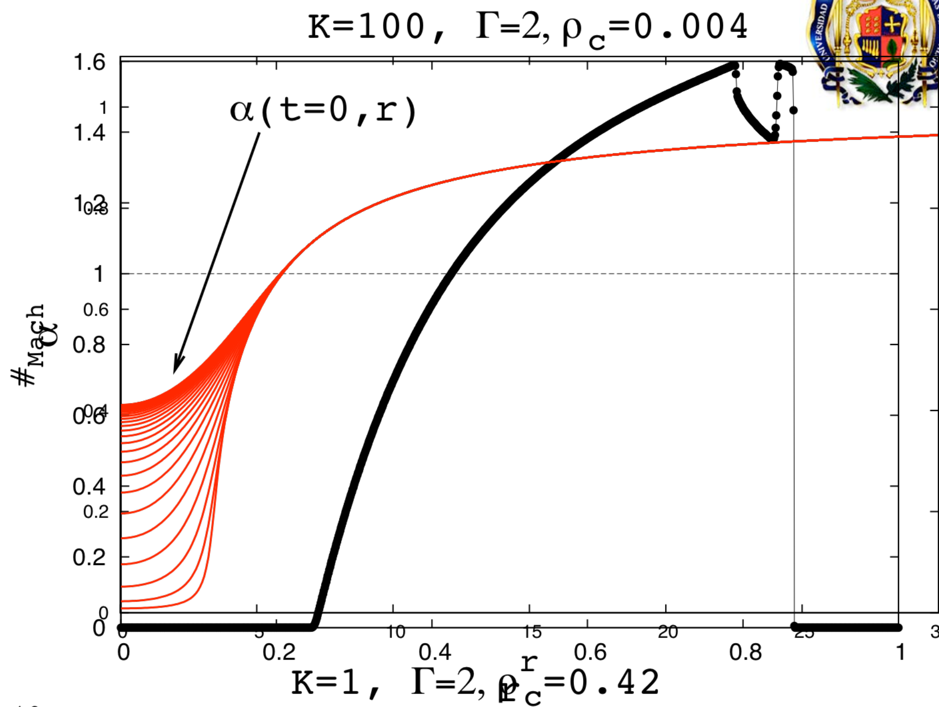
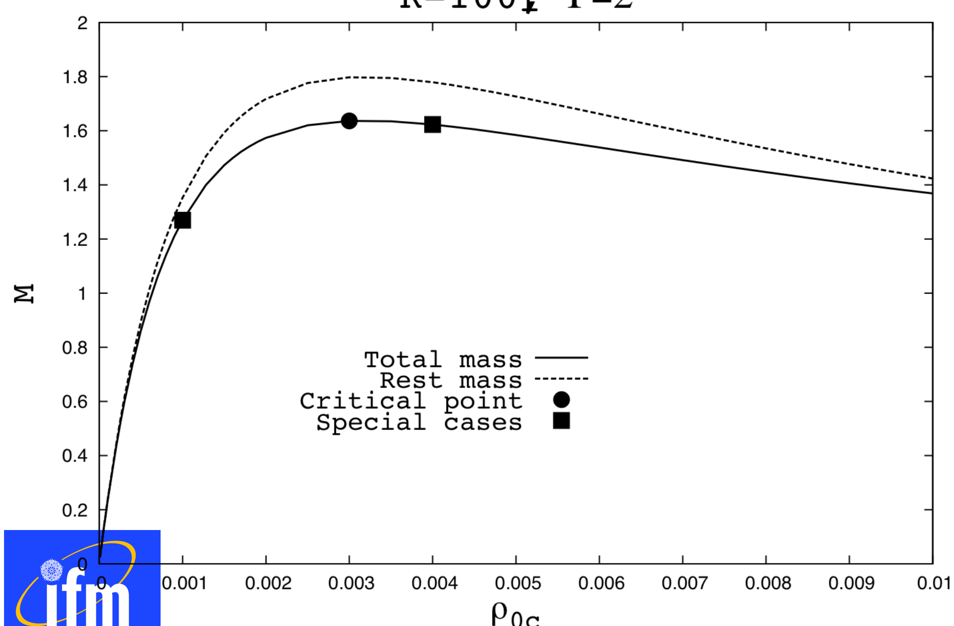
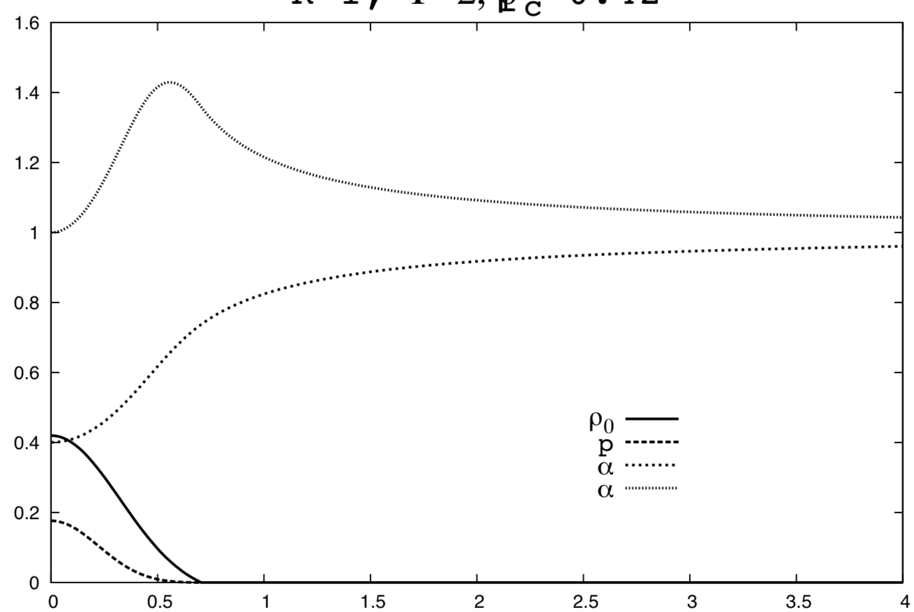
# Basic RHD

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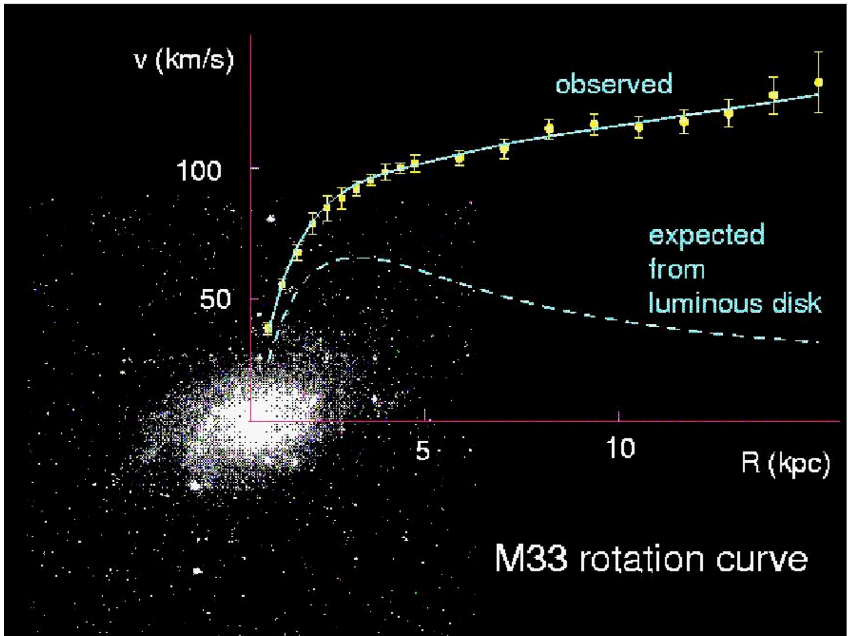
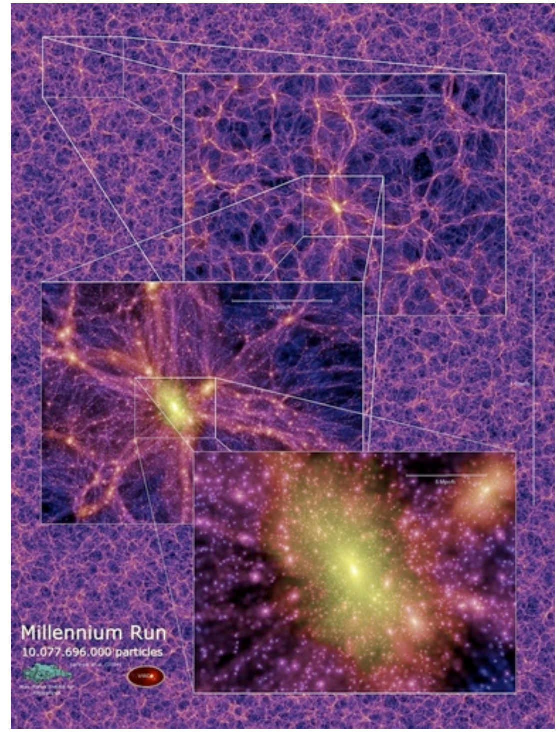
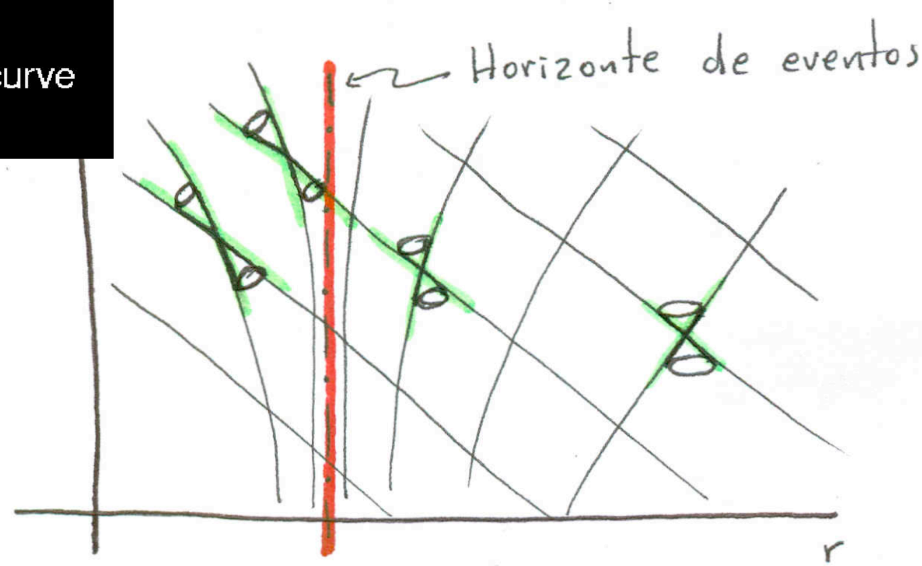
# Basic RHD in spherical symmetry



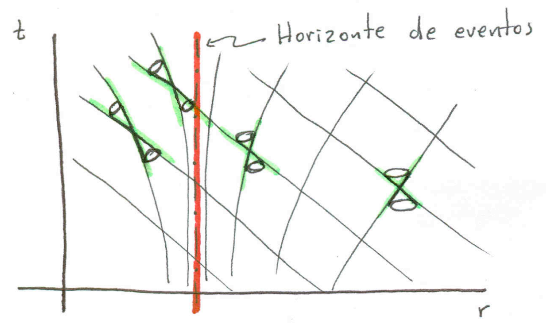

*h.mx*

# Numerical methods and initial conditions

Eddington-Finkelstein coordinates.

Excision.

$$p = (\Gamma - 1)\rho\varepsilon$$




Rather arbitrary initial set up:

- Start up
- Define velocity.
- Spatially constant density and various values of the velocity.

*One of the ideas is to investigate whether or not there is a late time attractor density – velocity – pressure profile. There is one.*

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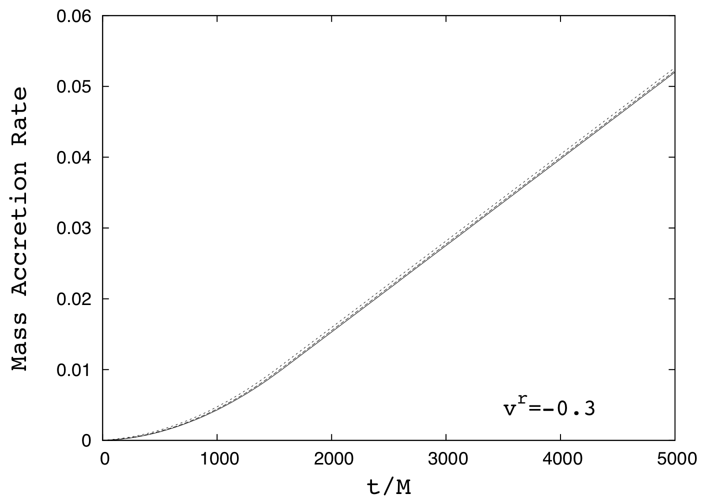
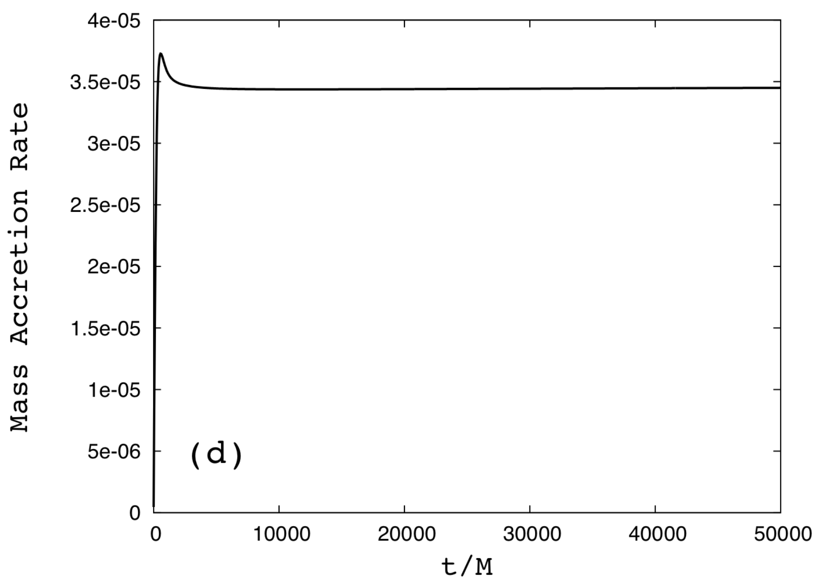
# Results

- For (p=0).




- For p > 0.

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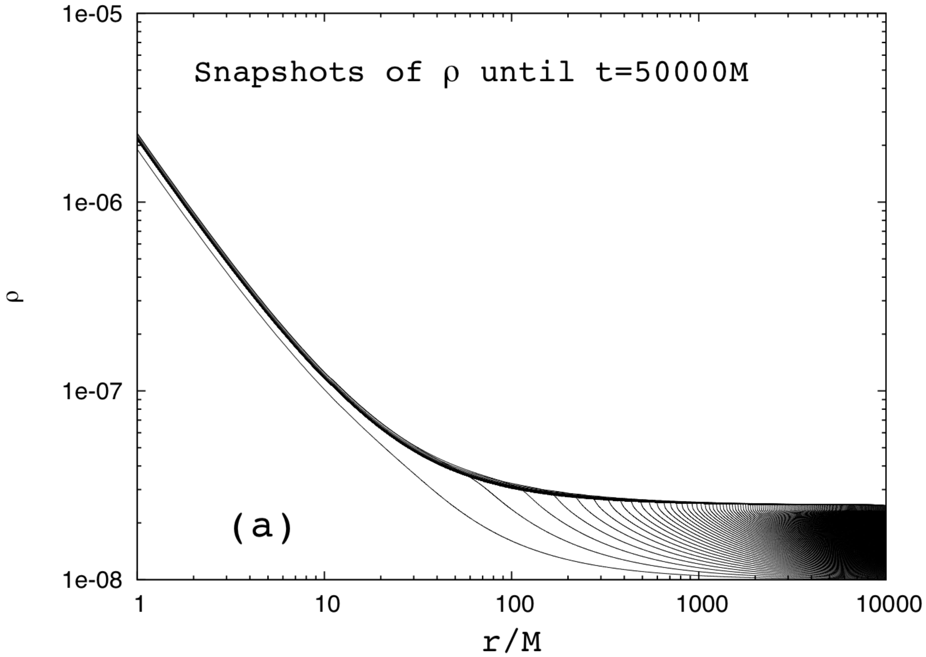
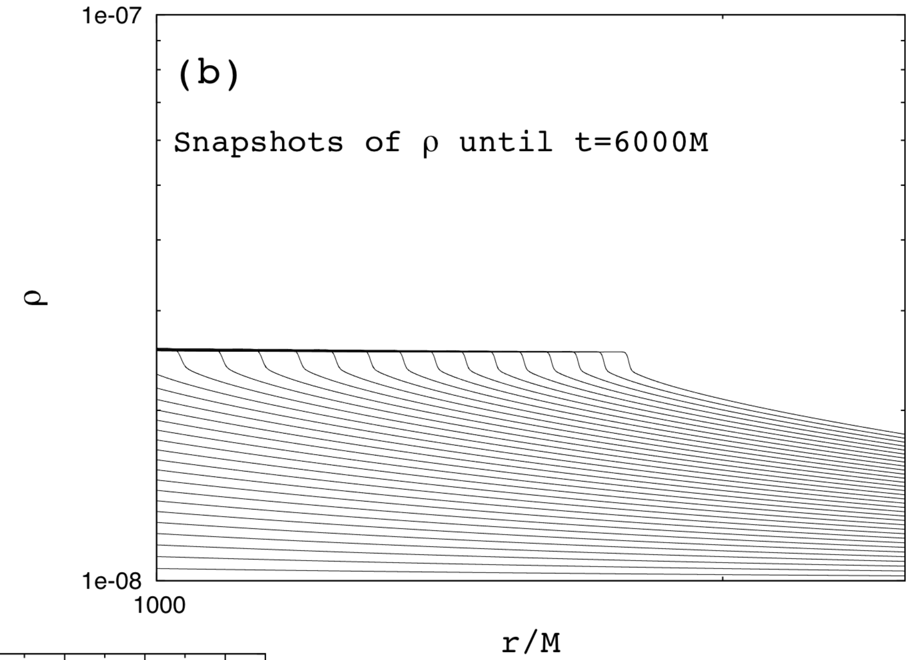
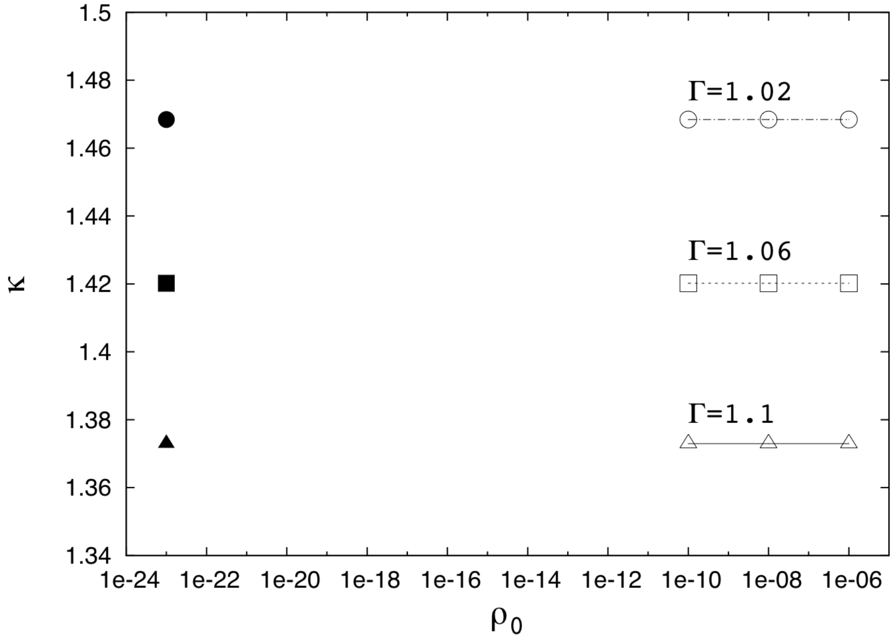
# Density profile




$$\rho = \frac{A}{r^{\kappa}}$$

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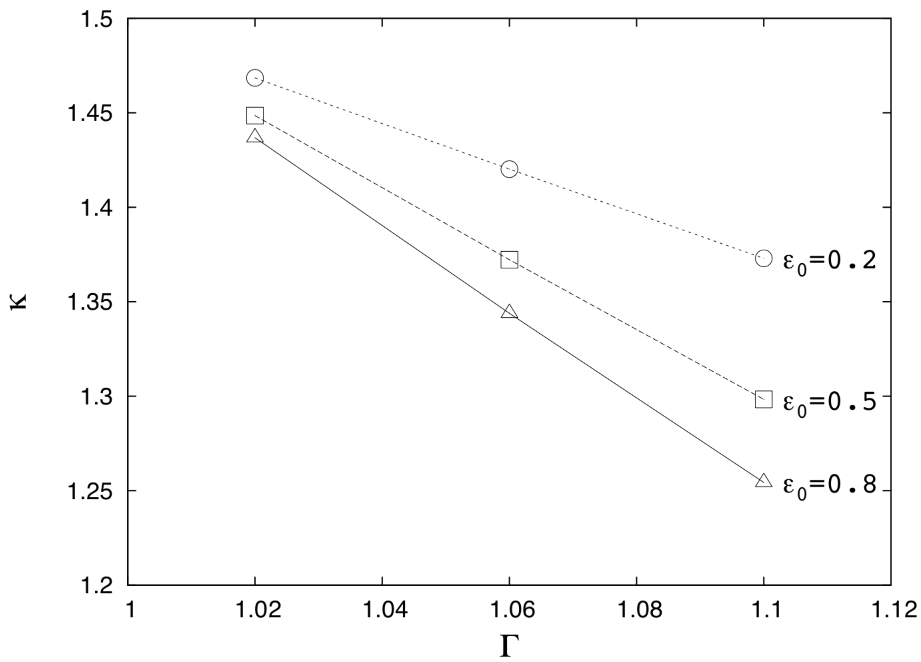
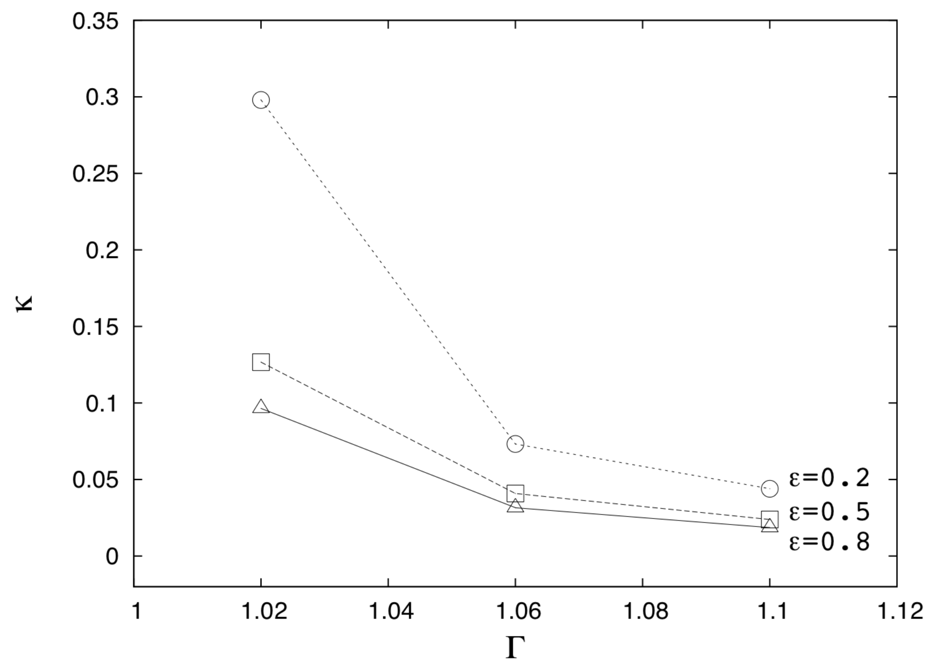
# Density profile

$$\rho = \frac{A}{r^{\kappa}}$$

NFW k->1…. Moore  k->1.5

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# In the end what we did on fixed background

1. We showed the runaway instability for p=0

2. We showed there are late-time attractor type solution for p>0

3. We showed that the amount of ideal gas accreted by SMBH seeds is a very small fraction of the seed's mass. That is, dark matter would not contribute to the SMBH mass significantly.

4. The density profile acquired by the gas is in no contradiction with cored dark matter halos.

*MNRAS **415**, (2011) 225-234.*
*MNRAS **416**, 3083–3088 (2011)*

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## Application to PBH growth

We study the PBH growth during the RDE

For this we solve NUMERICALLY the Einstein-Euler system for Spherical non-linear accretion of an ultrarelativistic gas with radiation ***Equation of State***

$$p = \rho / 3$$

on a NON-EXPANDING SCENARIO

*We track the apparent horizon growth* $\dot{M}$ *and then get a time function of the horizon growth.*

In order to simulate a scenario with expansion, we partition the time domain into small pieces in each of which we assume the condition of non-expansion holds.

This system is fed with the asymptotic value of density corresponding to that of radiation at a given cosmological time

$$\rho \sim 1/t^2 \qquad t \in [10^{-4}s, 100s]$$

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# Application to PBH growth

We use the partition the time interval

$$t \in [t_0, t_f] = [10^{-4} s, 100 s]$$

$$t_0 < t_1 < \ldots < t_{N-1} < t_f$$

Start with an already formed PBH with mass $M_0$

Then knowing the mass growth rate $\dot{M}$ after the first interval one has

$$M_{i+1} = M_i + \dot{M}(t_{i+1} - t_i)$$

And can be integrated up to the end of the desired time window.

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# More on PBH growth

1. We get a linear growth $\Delta M = \dot{M}\Delta t$ with $\dot{M}$ constant.
   - We use ADM, excision, FD, constraint preserving BCs.

2. We use it to estimate how much a BH grows in a small interval with a given asymptotic density.

   - Start with an initial value of the PBH mass and use an asymptotic radiation density at that exact initial time in the cosmological history.
   - Use $\dot{M}$ to estimate a **final BH mass** in a little time interval.
   - Update the asymptotic radiation density and calculate a new $\dot{M}$
   - Use it to calculate a new final **BH mass**

3. We repeat the process until we cover the entire desired time window

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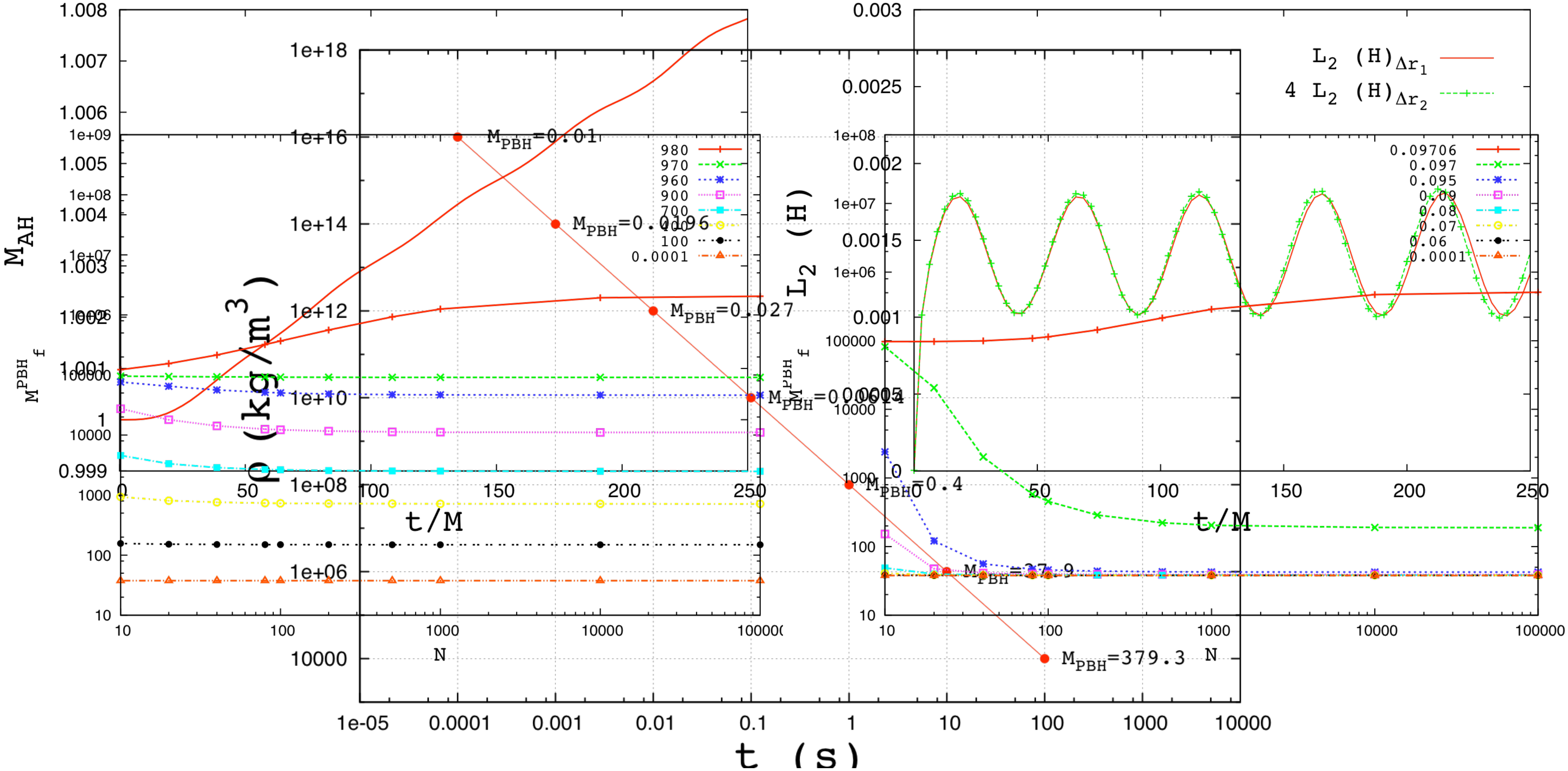
# More on PBH growth

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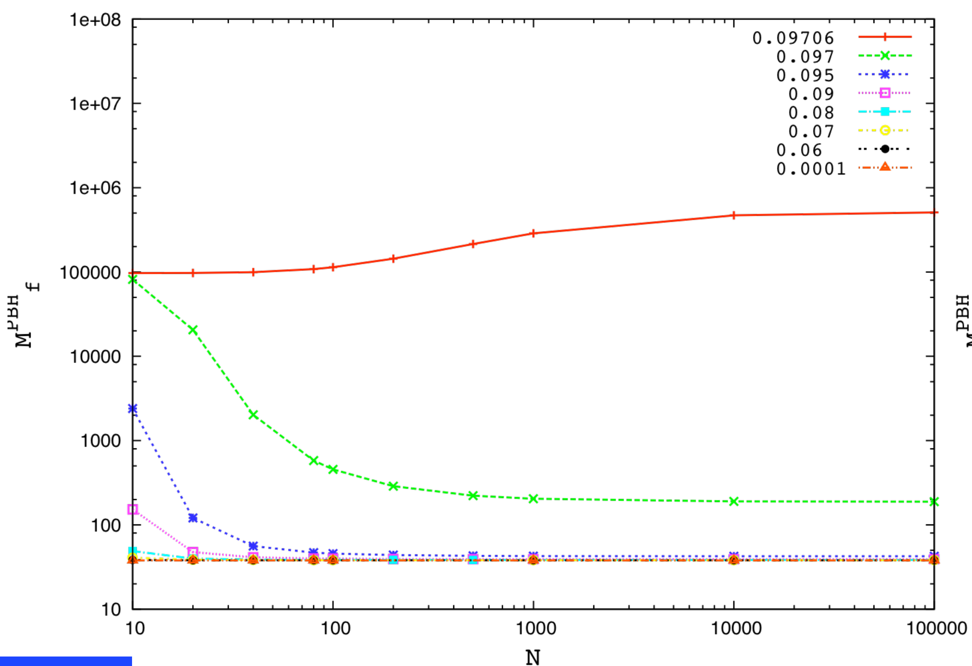
# More on PBH growth [Holm 15A: MBH~10^10SM]

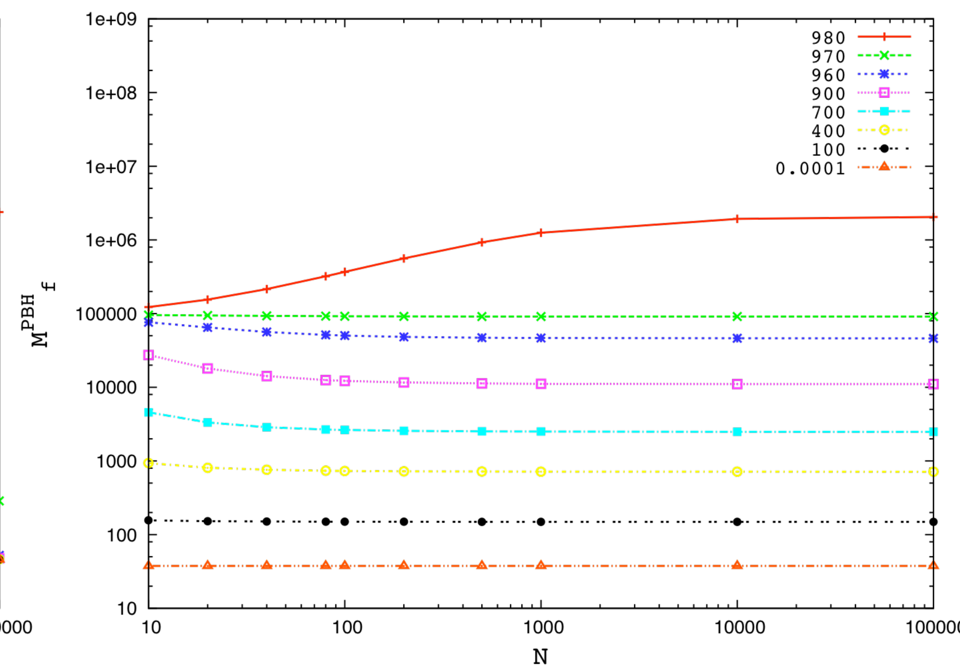
Case 1. The time window is $t \in [10^{-4}s, 100s]$, the PBH is assumed to be formed at $t = 10^{-4}s$

Case 2. The time window is $t \in [1s, 100s]$, the PBH is assumed to be formed at $t = 1s$

$M_0^{PBH} \in [10^{-4}, 0.095] M_{\otimes}$




$M_0^{PBH} \in [10^{-4}, 970] M_{\otimes}$

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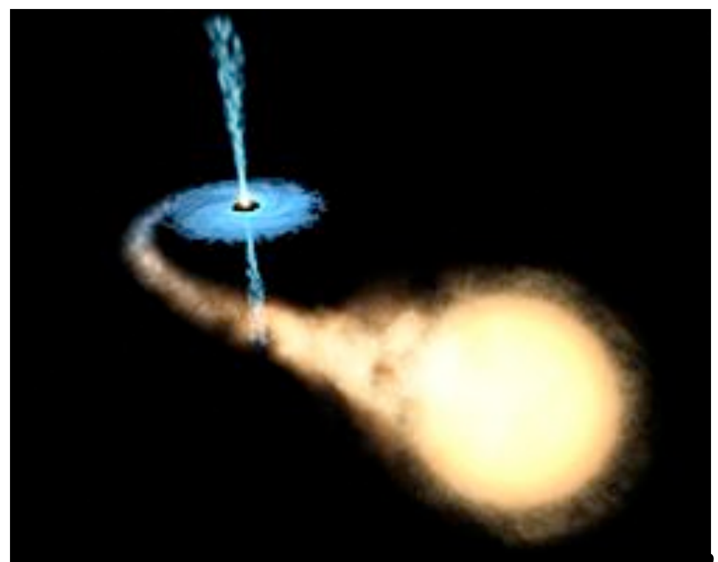
# RHD: near high energy astrophysics

Gamma ray burts (GRBs)

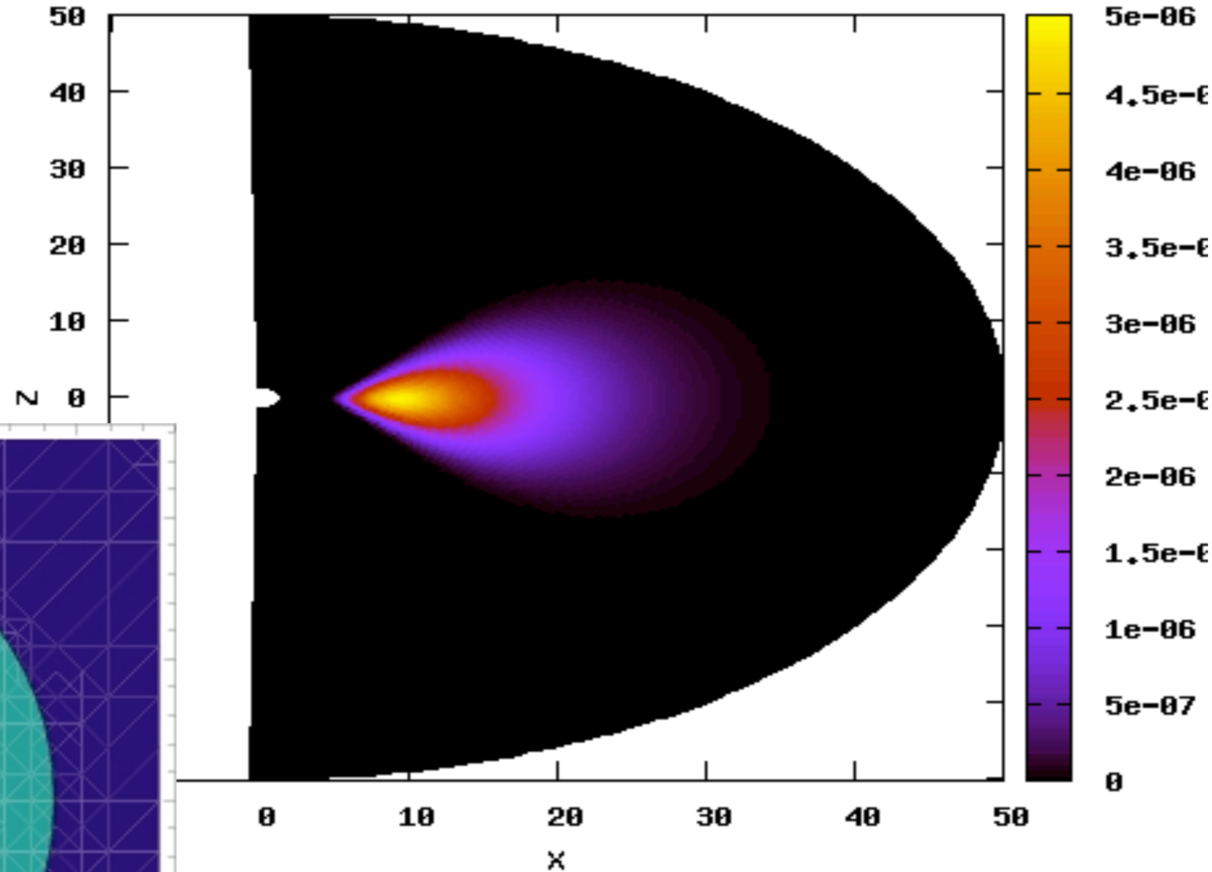
X-ray sources & QPOs

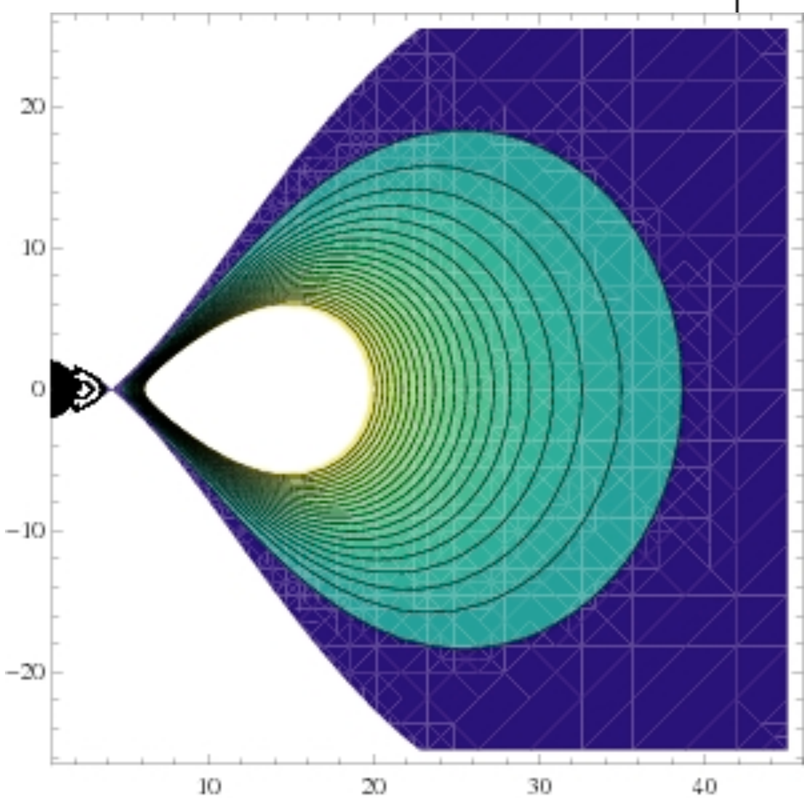
These have in common that the source is related yo STRONG gravitational fields and important magnetic fields

It has become an excellent
Opportunity for those working on
Numerical relativity to try to enter
The astrophysics community.



# Tori

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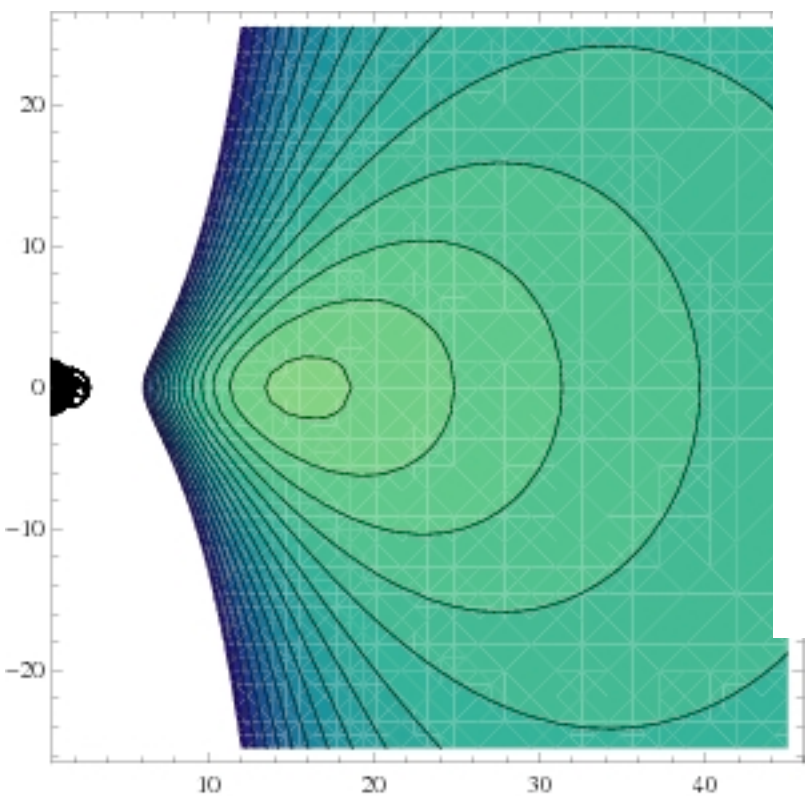
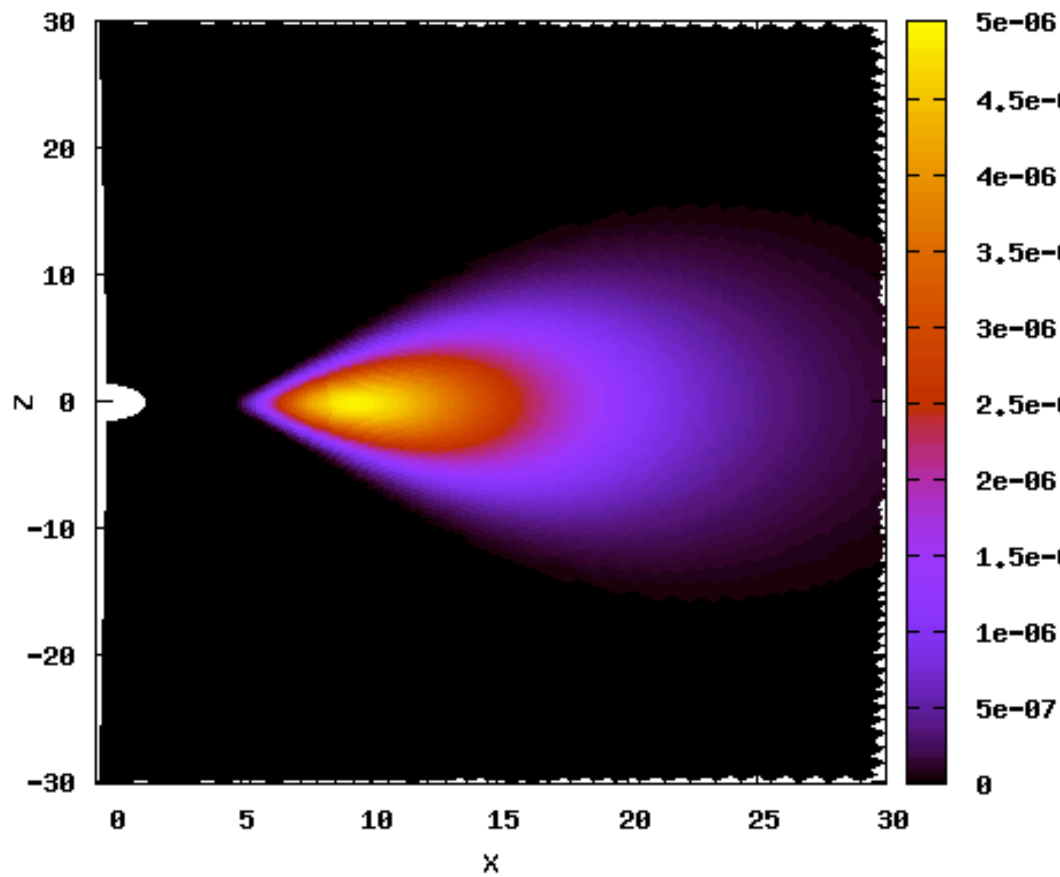
# Tori

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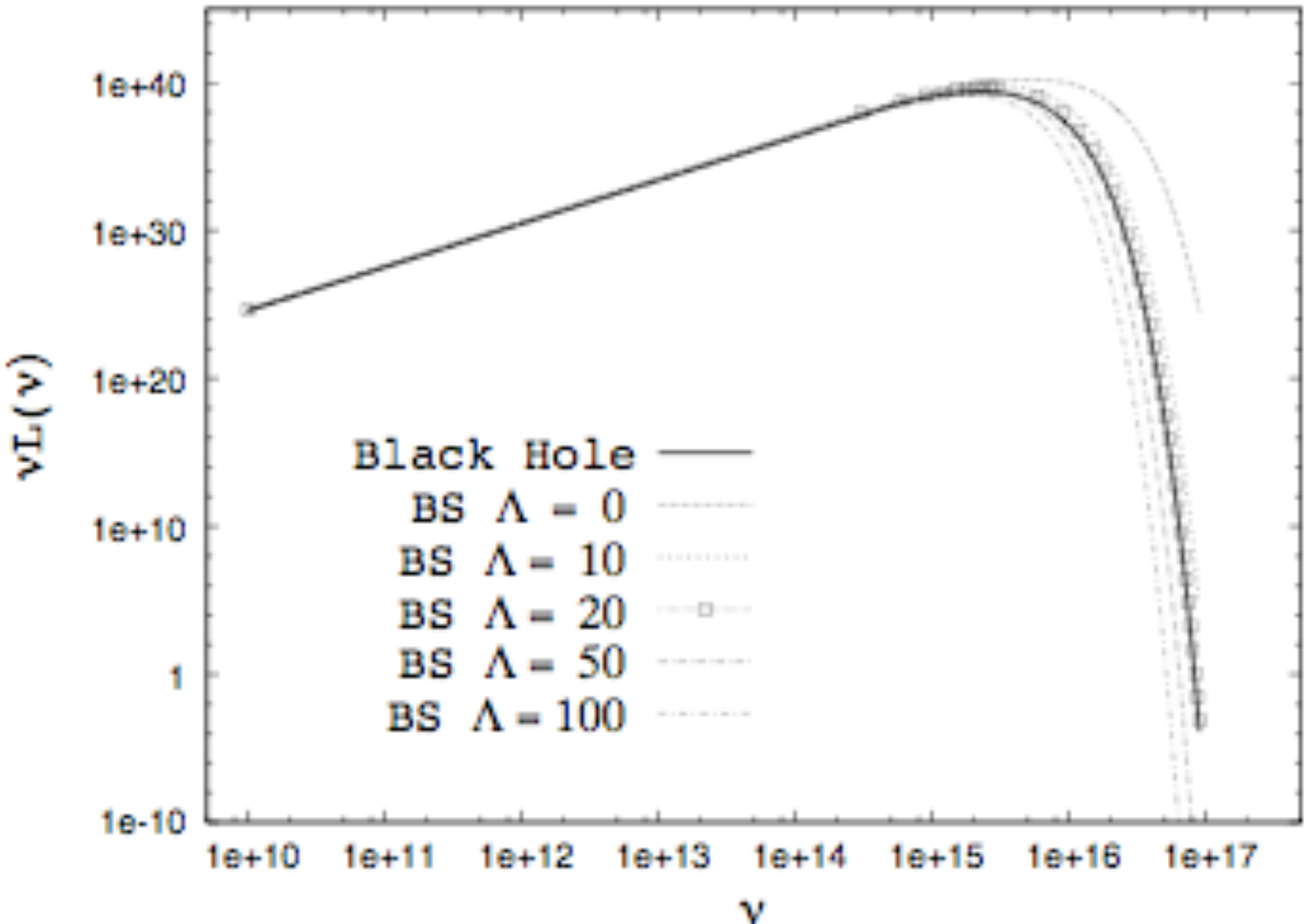
# What is this?





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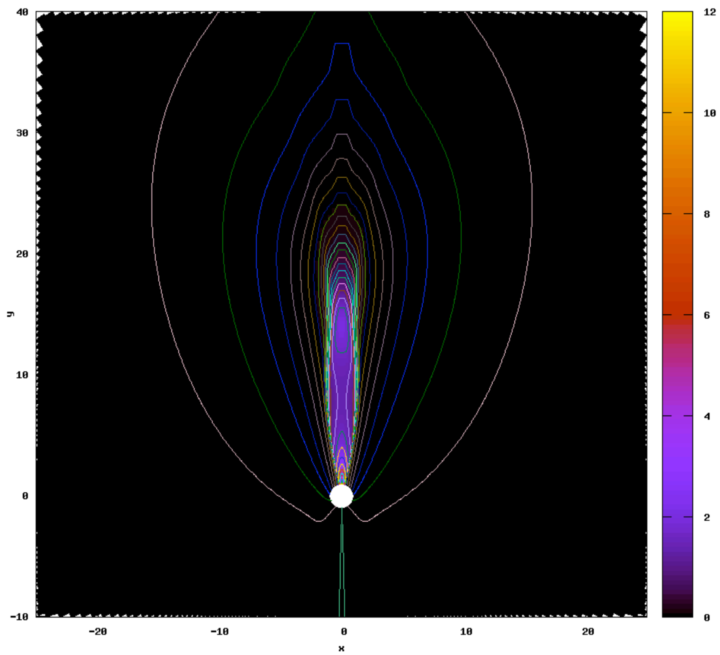
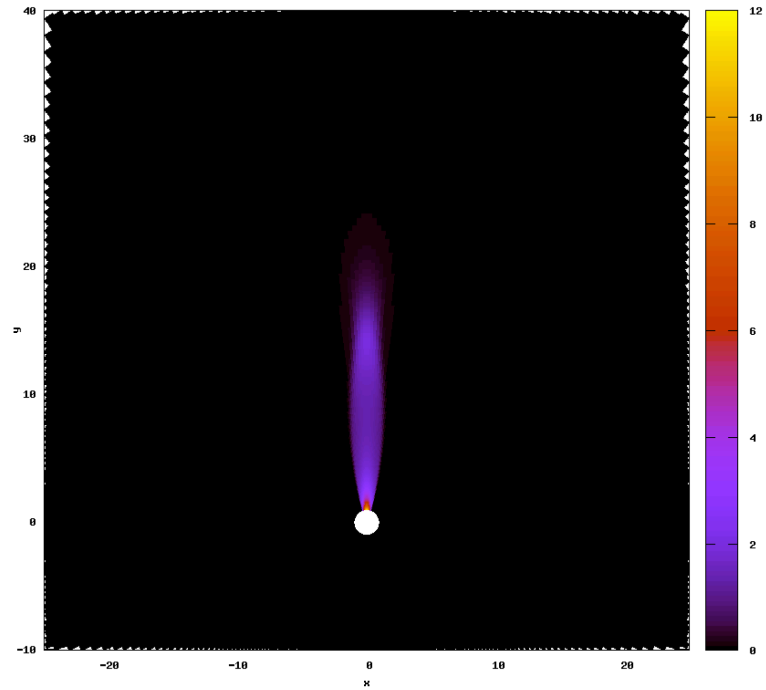
# Bondi-Hoyle accretion

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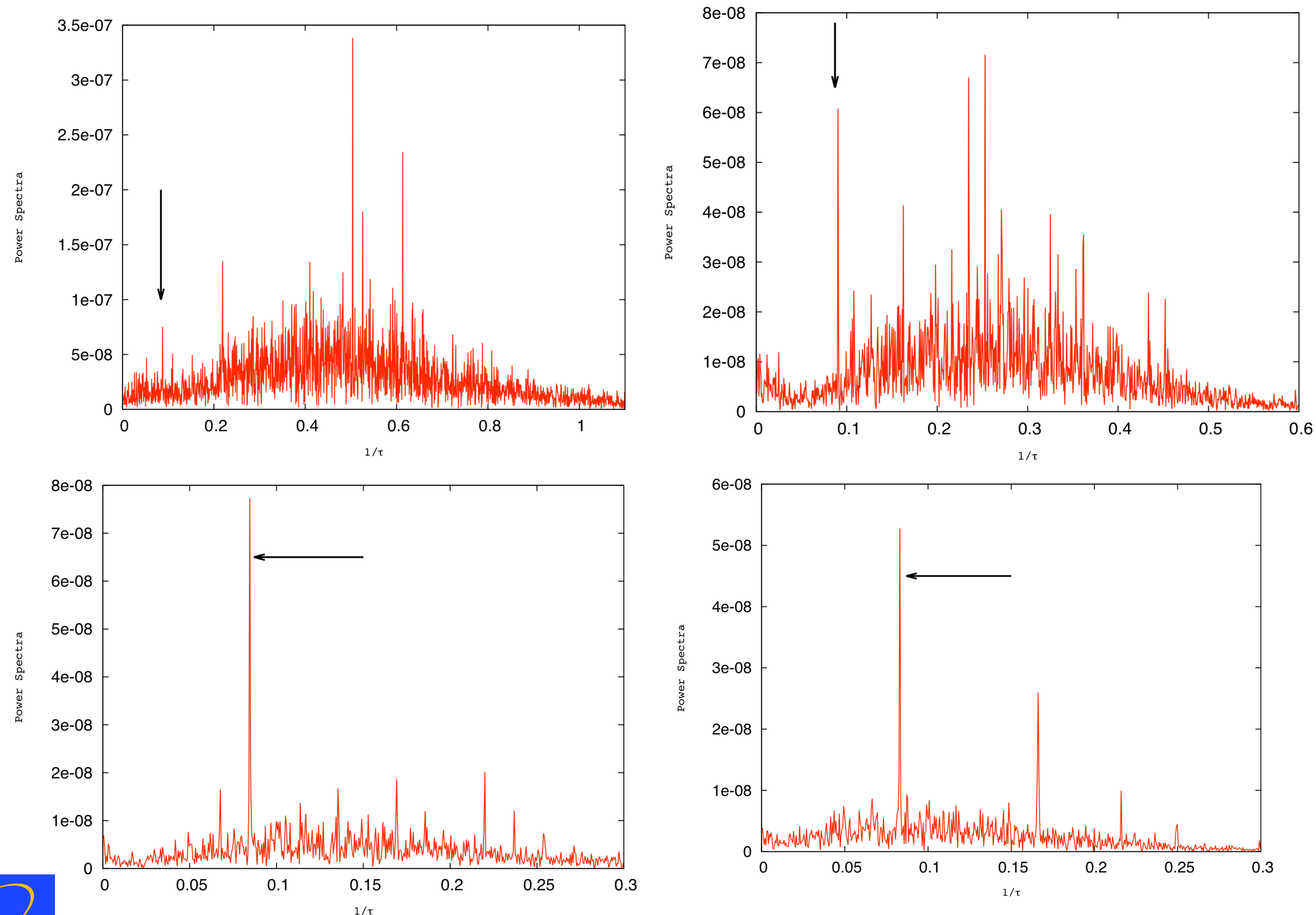
# Winds

As the shock cone vibrates one can ask whether there are global vibration modes.

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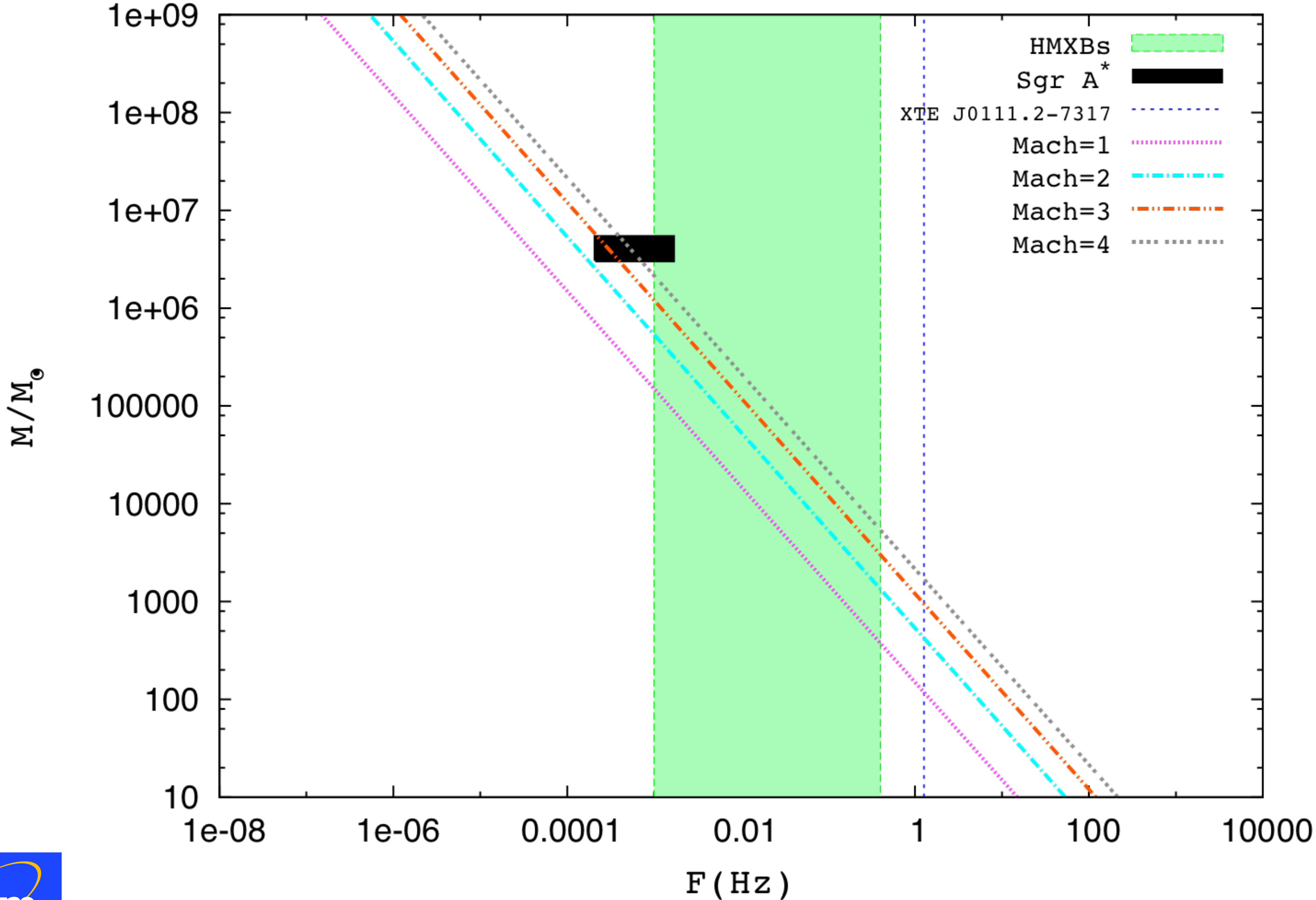


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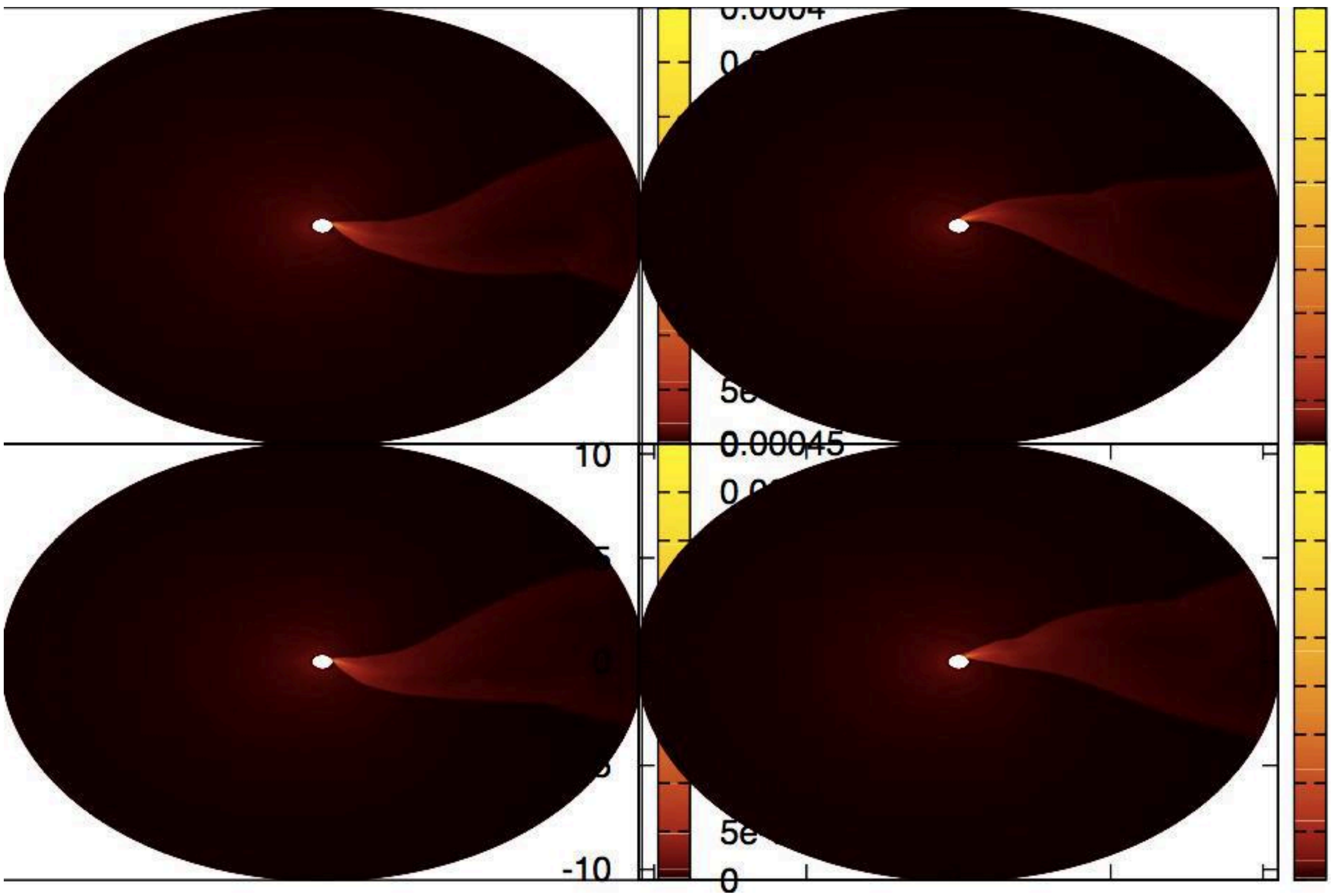
# Flip flop instability

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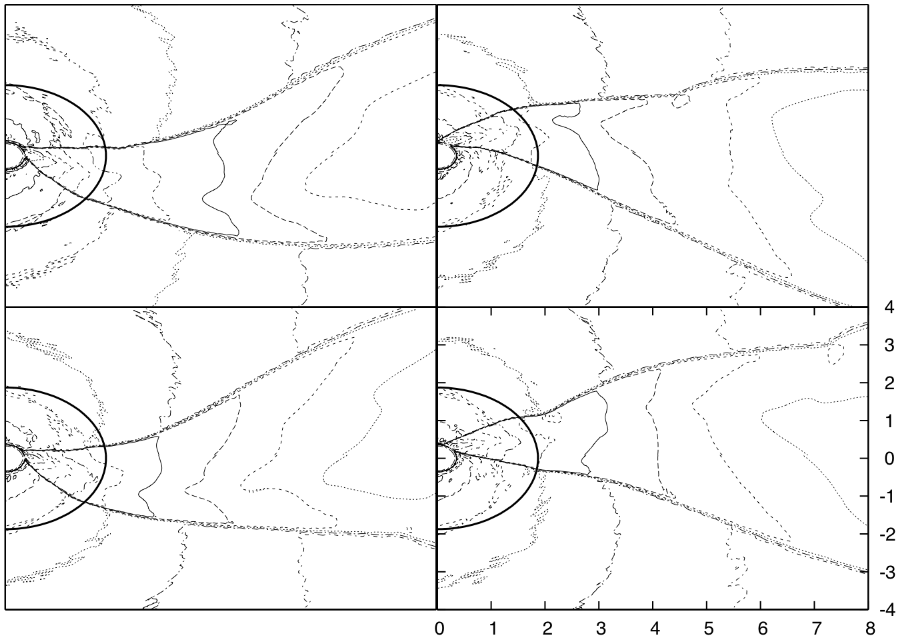
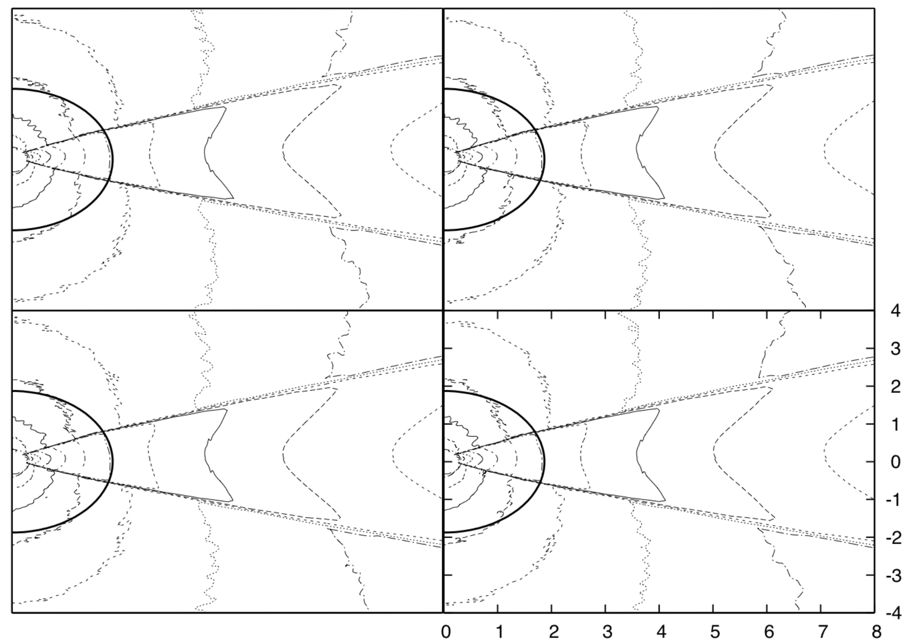
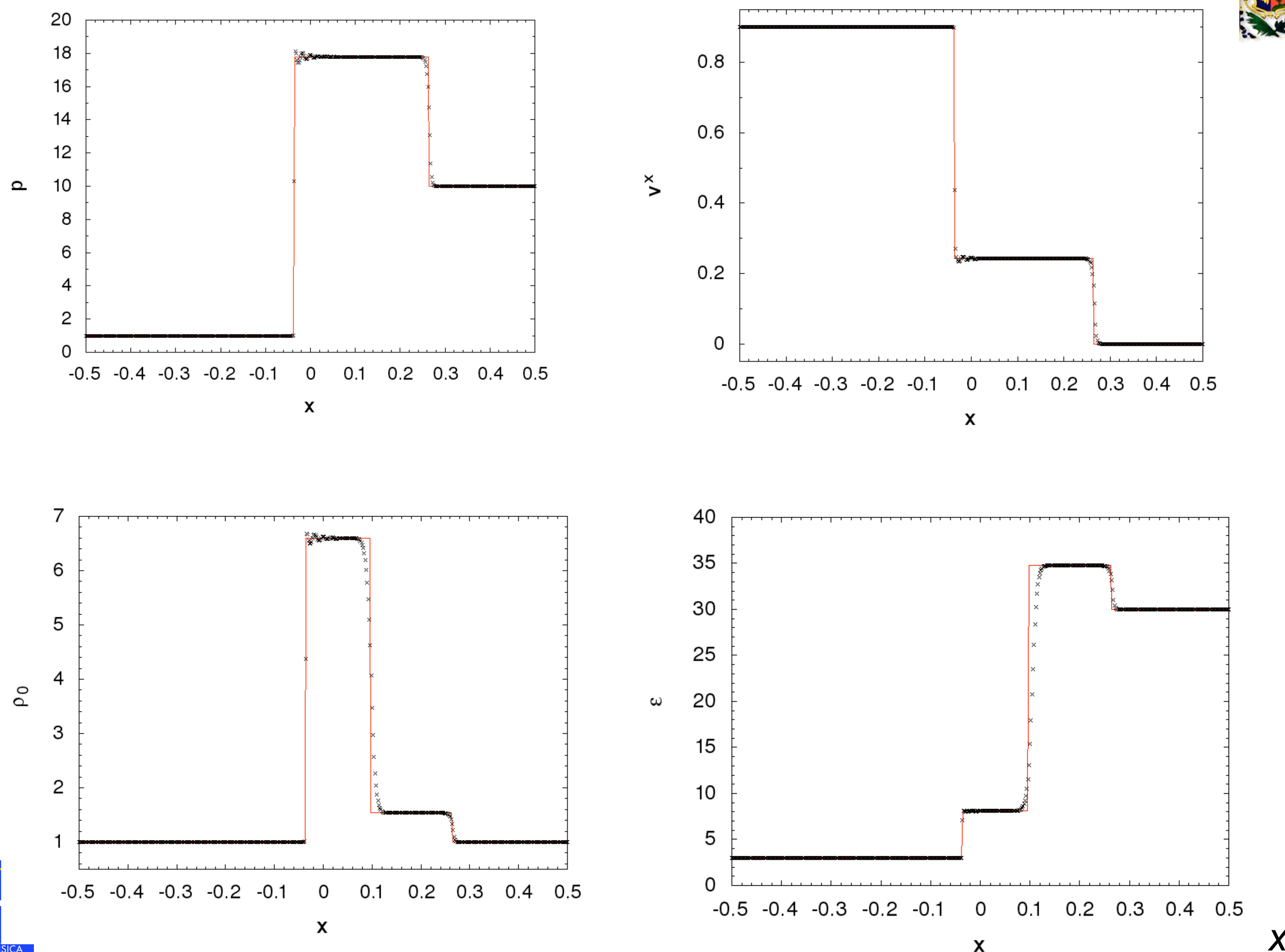


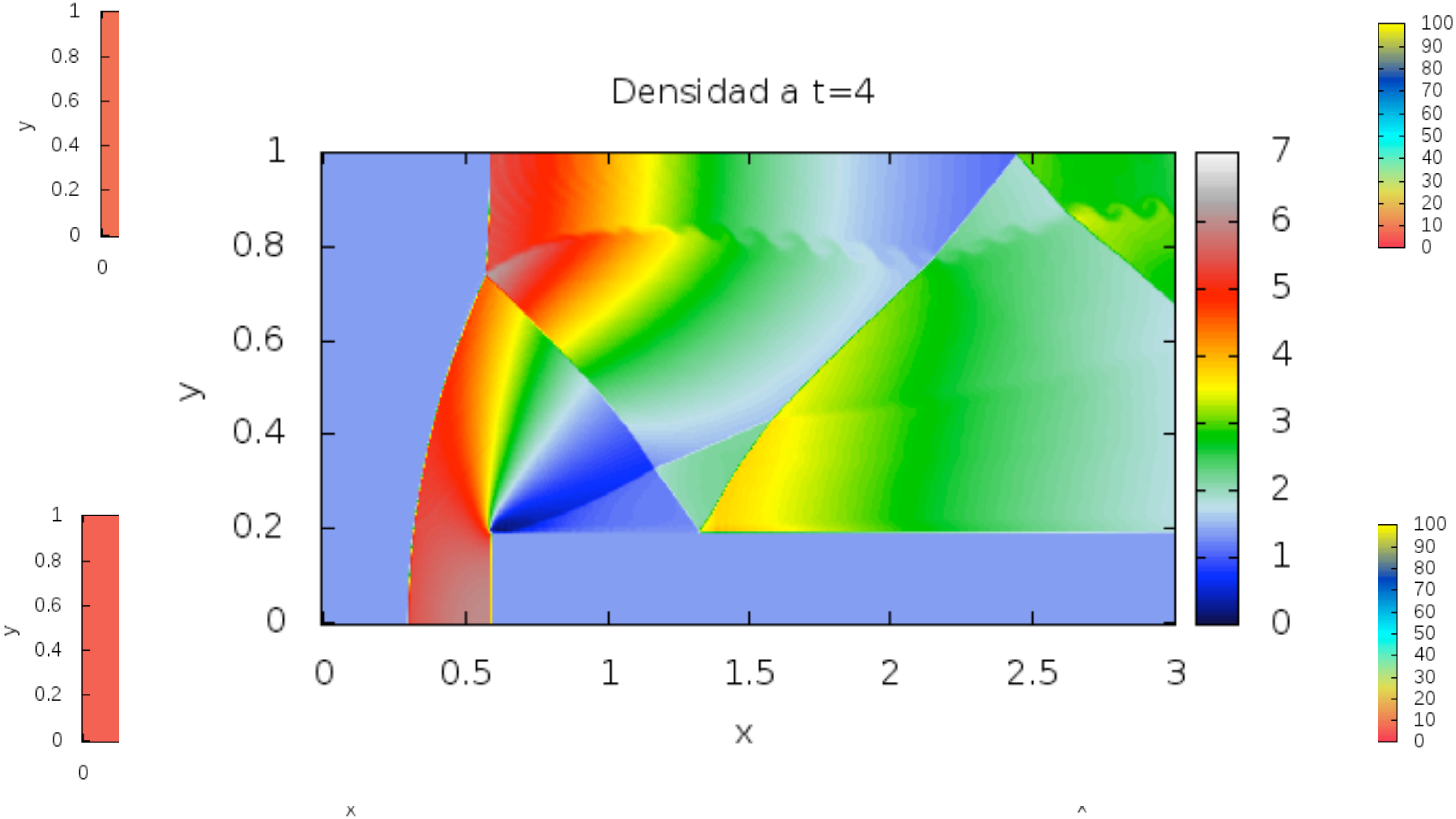




*MNRAS **429**, (2013) 3144-3154*
*MNRAS **426**, (2012) 732–738*

# The RHD 3D status



# The RHD 3D (howto)

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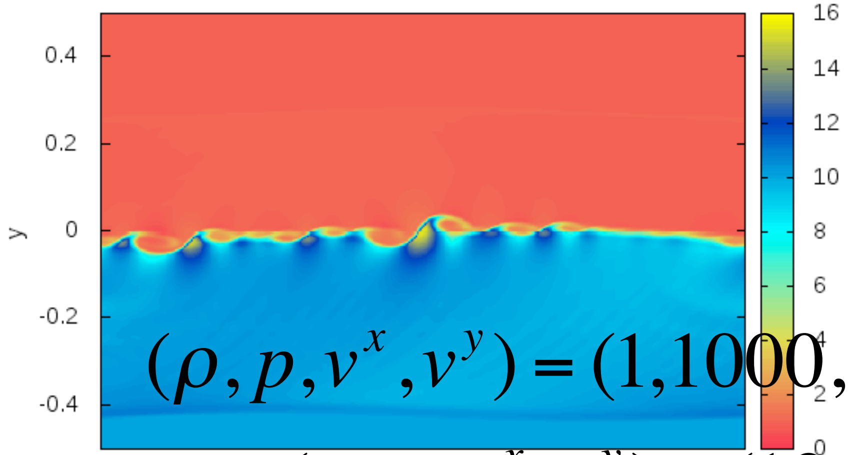
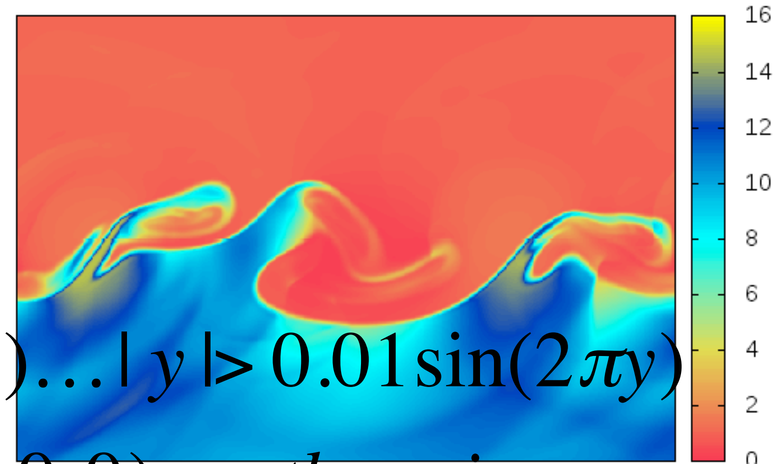
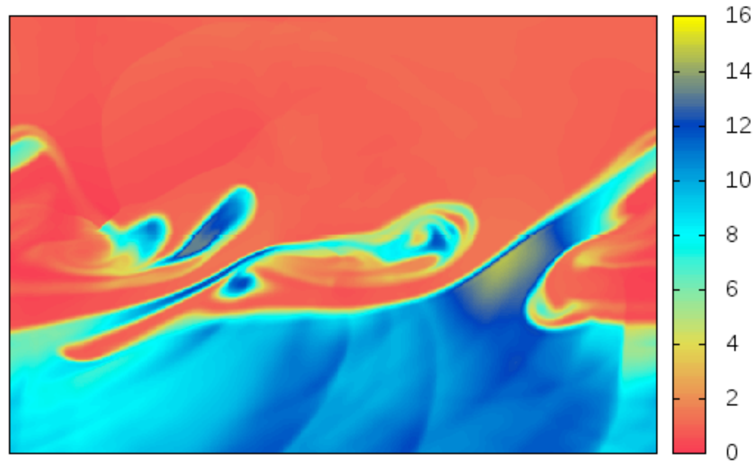
# The RHD 3D status

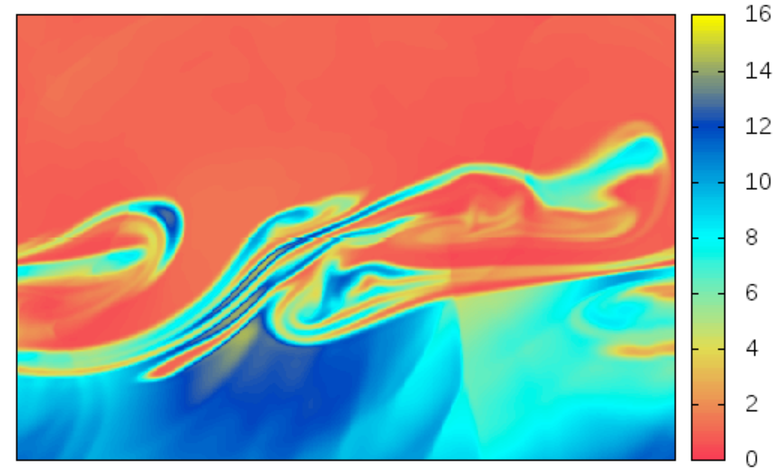



$$(\rho, p, v^x, v^y) = (1, 1000, 0, 0.9) \ldots |y| > 0.01 \sin(2\pi y)$$

$$(\rho, p, v^x, v^y) = (10, 1000, 0, 0) \ldots otherwise$$



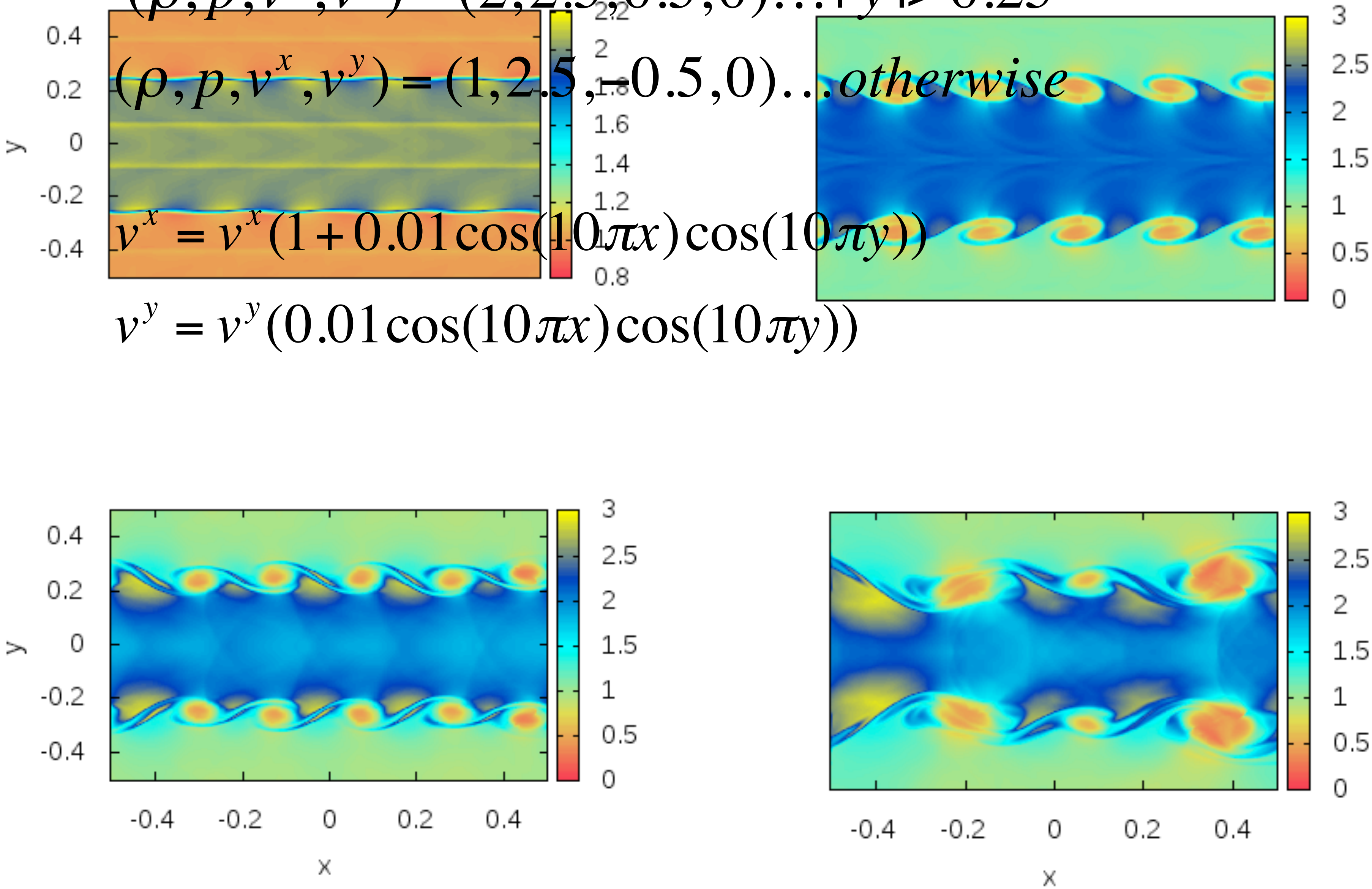
# The RHD 3D status

$$(\rho, p, v^x, v^y) = (2, 2.5, 0.5, 0) \ldots |y| > 0.25$$

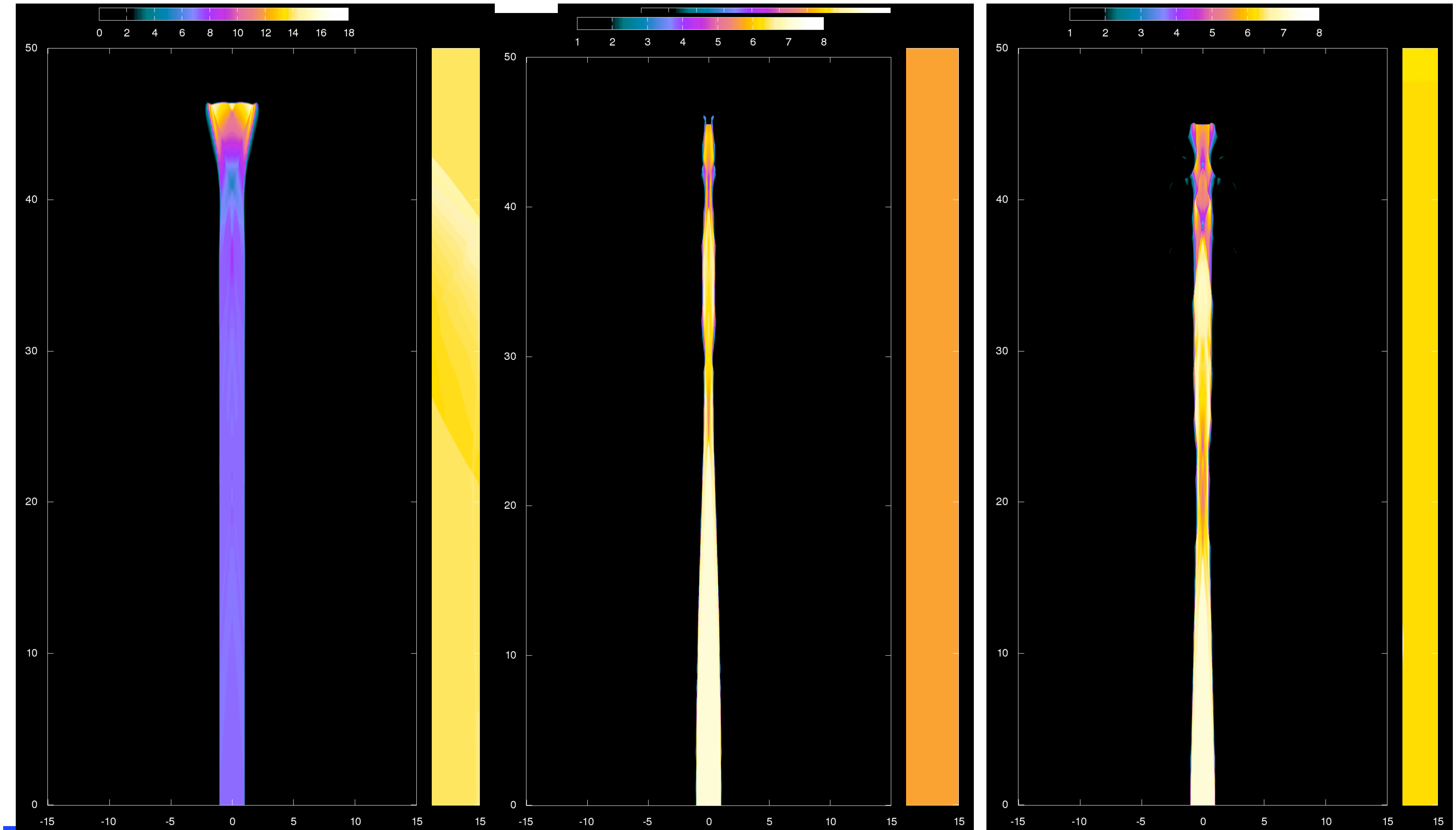
$$(\rho, p, v^x, v^y) = (1, 2.5, -0.5, 0) \ldots otherwise$$

$$v^x = v^x(1 + 0.01\cos(10\pi x)\cos(10\pi y))$$

$$v^y = v^y(0.01\cos(10\pi x)\cos(10\pi y))$$



# The RHD 3D status

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# Relativistic Magnetohydrodynamics (Fabio & Alex)

$$\nabla_\nu(\rho_0 u^\nu) = 0$$

$$\nabla_\nu(T^{\mu\nu}) = 0$$

$$\nabla_\nu({}^*F^{\mu\nu}) = 0$$

$$b^\mu = u_\nu {}^*F^{\mu\nu} \quad h = 1 + \varepsilon + p/\rho_0$$

$$p^* = p + p_m = p + b^2/2$$

$$h^* = h + h_m = h + b^2/\rho_0$$

$$T^{\mu\nu} = \rho_0 h^* u^\mu u^\nu + p^* g^{\mu\nu} - b^\mu b^\nu$$

$$\partial_0\left(\sqrt{\gamma}\vec{F}^0\right) + \partial_i\left(\sqrt{\gamma}\vec{F}^i\right) = \sqrt{-g}\vec{S}$$

$$\nabla\cdot\vec{B} = \frac{1}{\sqrt{\gamma}}\partial_i\left(\sqrt{\gamma}BB^i\right) = 0$$

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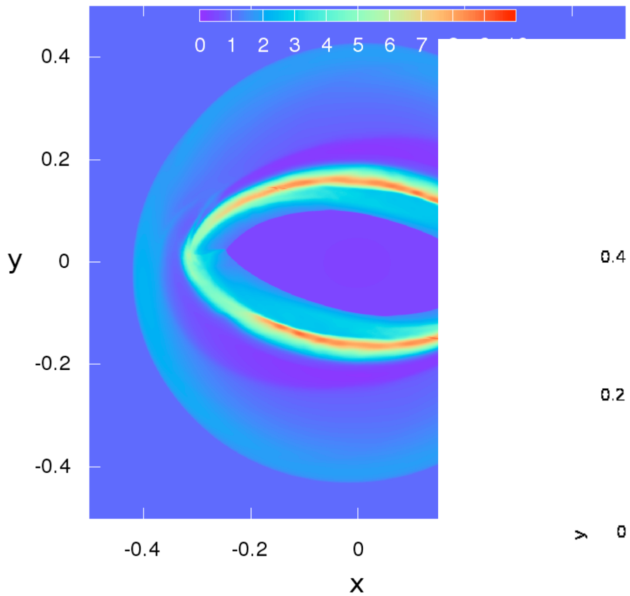
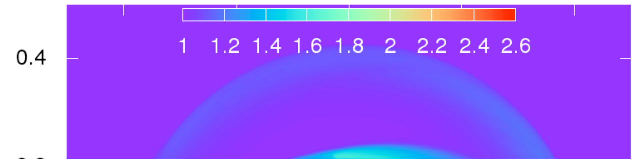
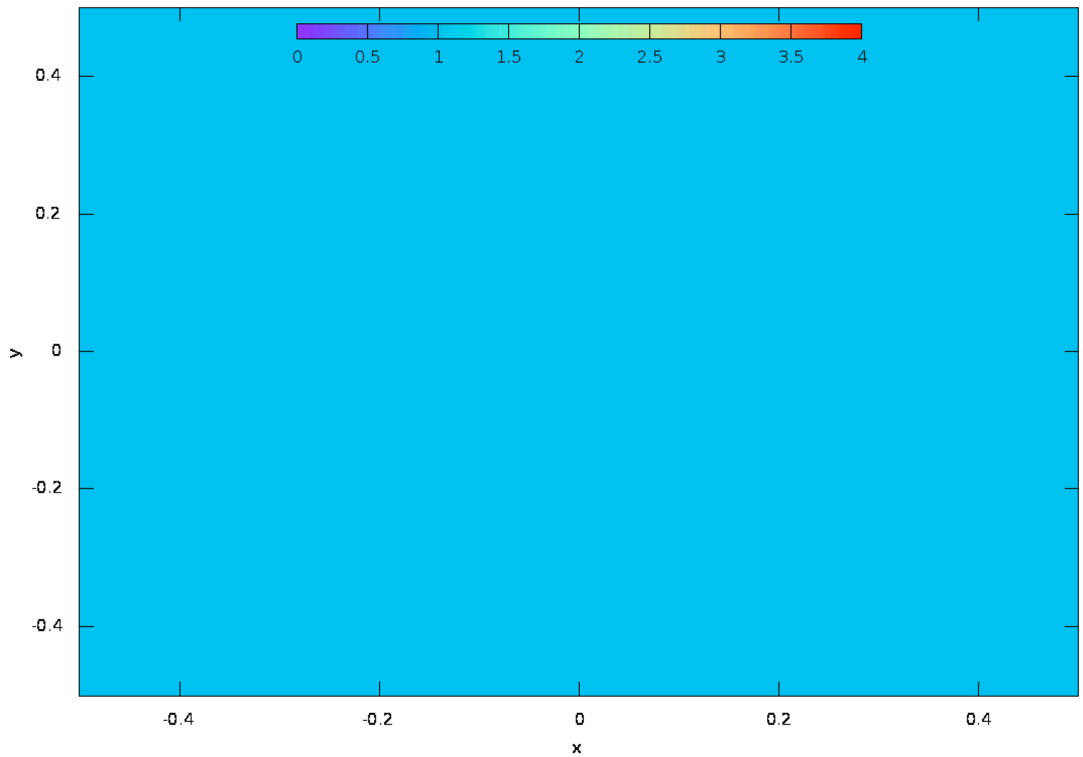
# Magnetic rotor




$\rho = 1$

# Cylindrical explosion test

This is also a cylindrical shell of radius $r_c = 0.8$ within which

$\rho = 0.01$

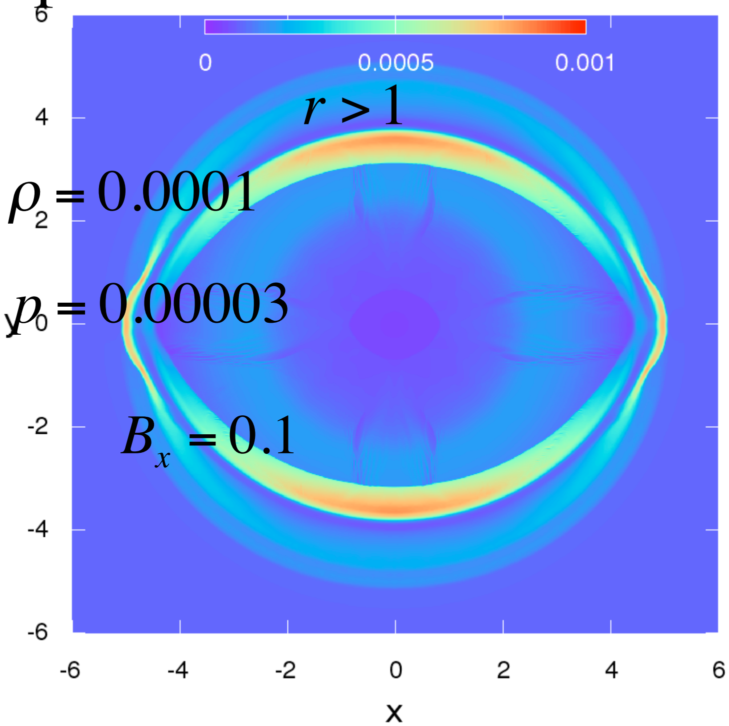
$p = 1$

$r > 1$

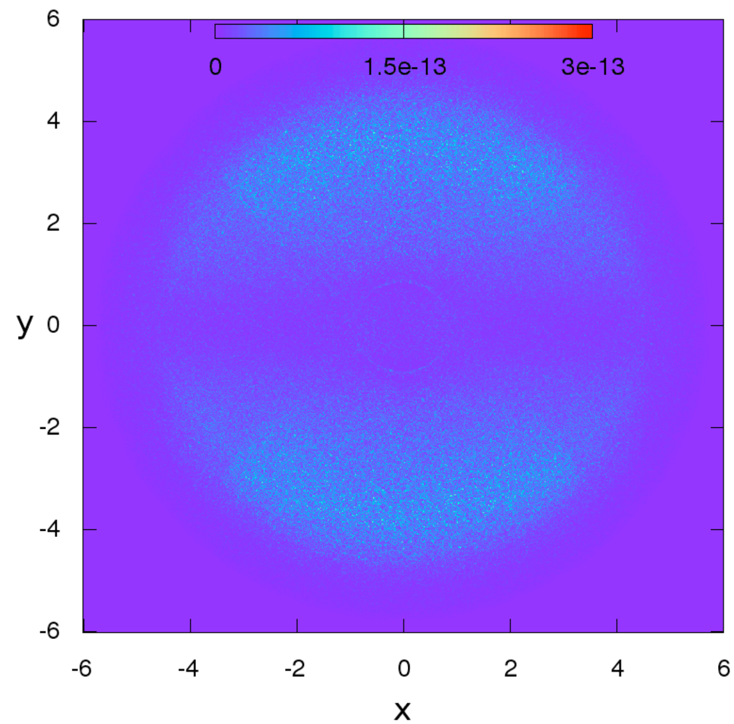
$\rho_2 = 0.0001$

$p = 0.00003$

$B_x = 0.1$




$r > 1$

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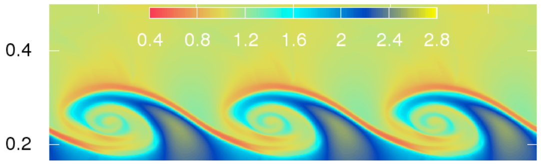
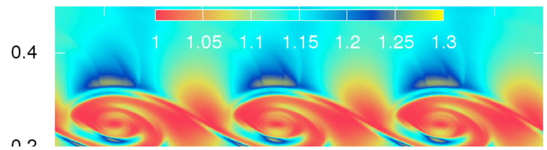
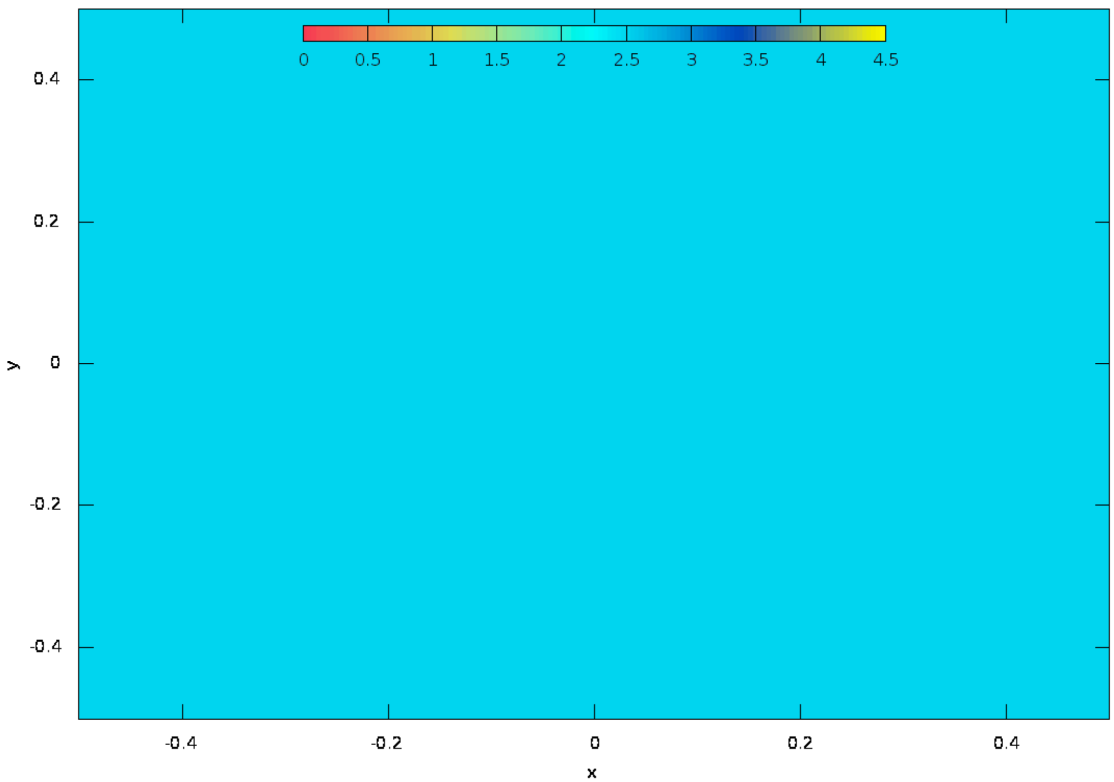
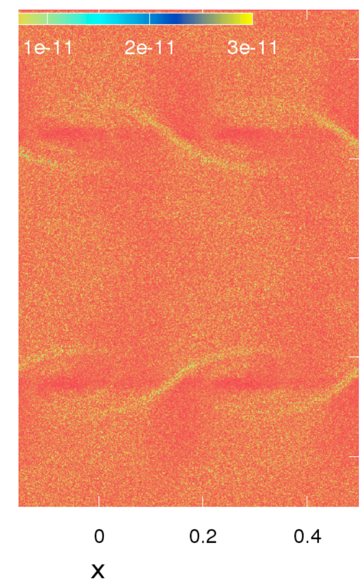
# More Kelvin-Helmholtz instabilities



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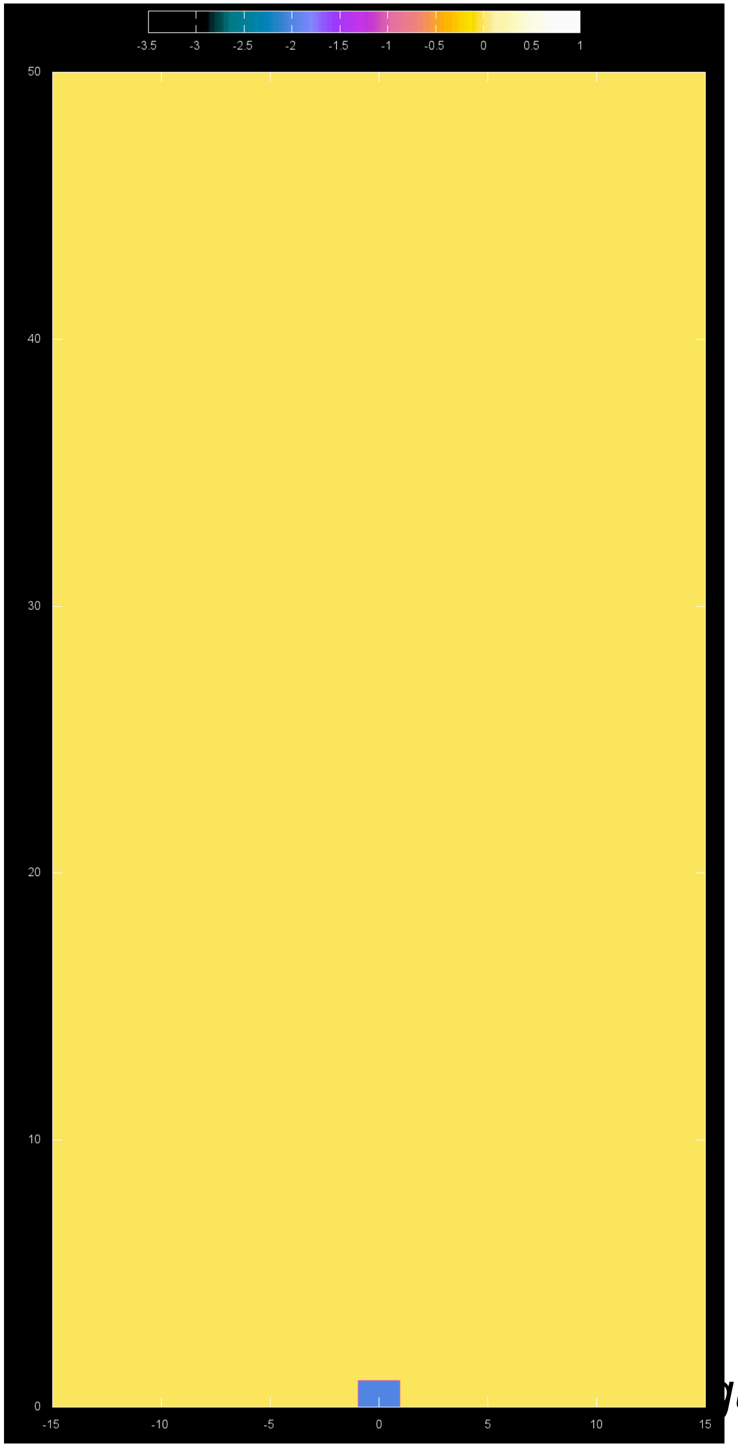


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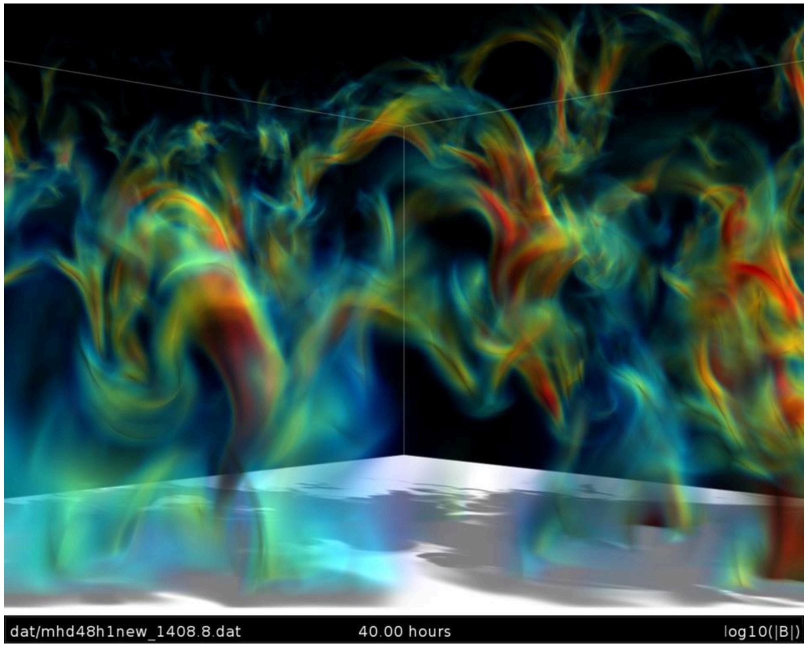
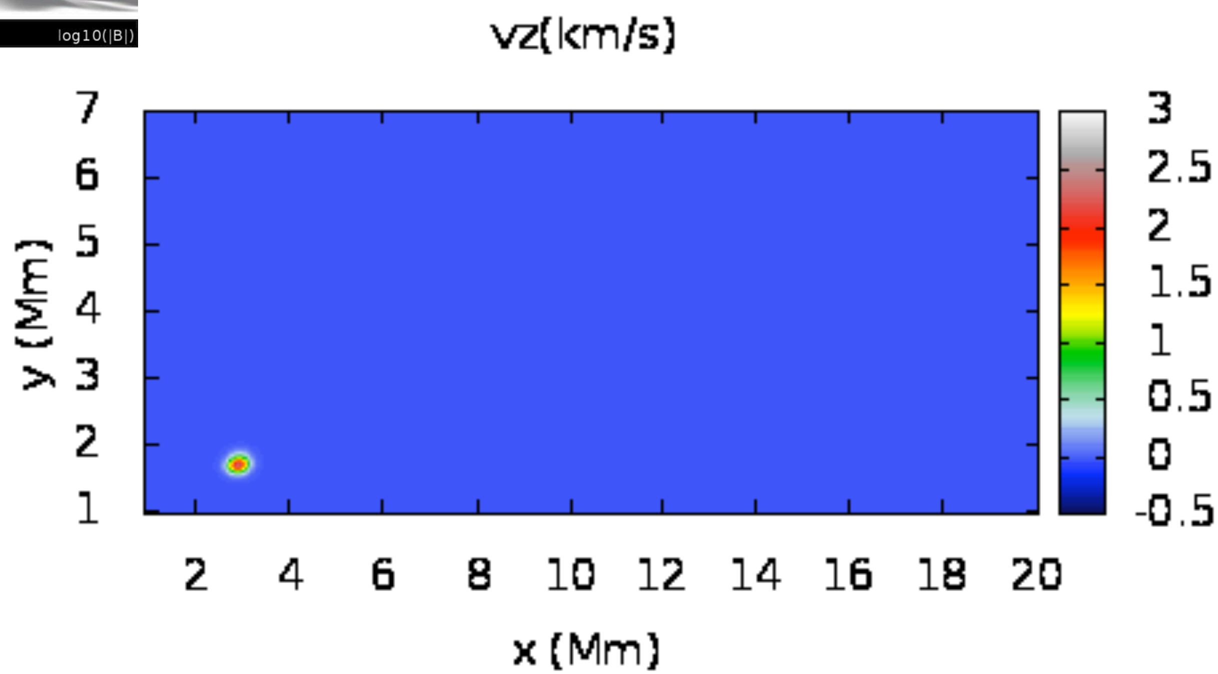
# We are also Newtonian (Juan & Francisco)

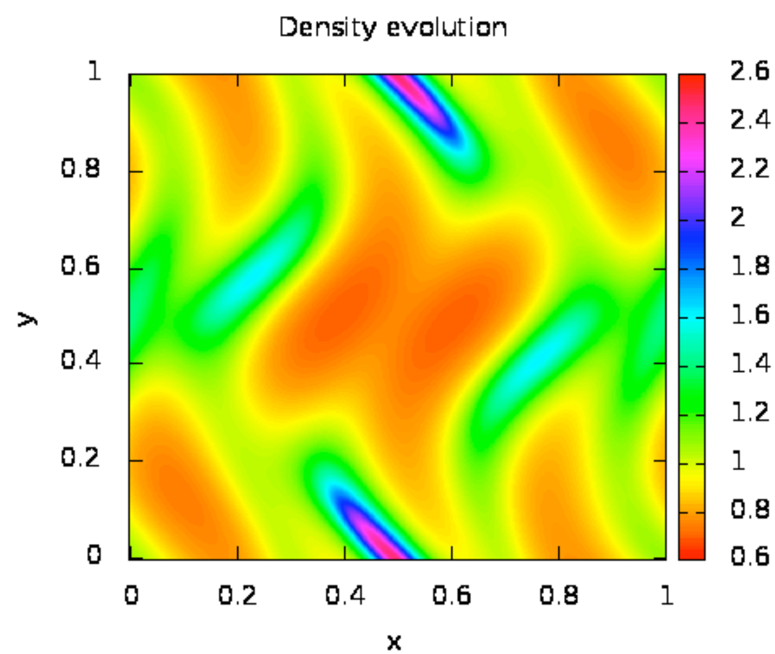


# We are also Newtonian

As we speak, our codes are running with radiation.

This requires a sophistication hard to handle with codes in the market.

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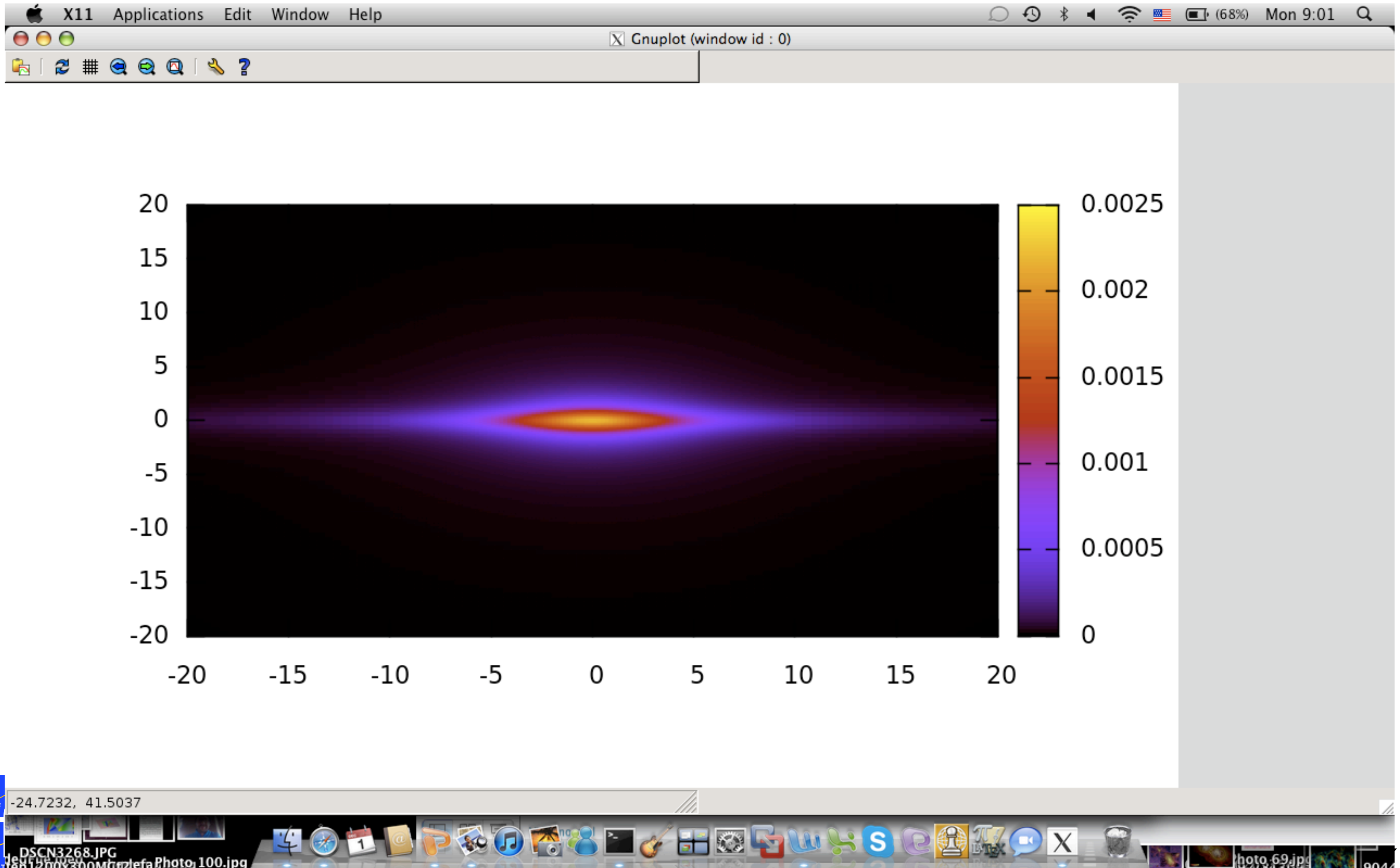
# We are also Newtonian (GPP-Euler (another example))



# Moral on numerical methods (Liz Bradley)

- Numerics can cause distortions, bifurcations, etc
- The results look a lot like REAL, PHYSICAL dynamics
- Source: algorithms, arithmetic system, timestep, etc
- Q: what could you do to diagnose whether your results included spurious numerical dynamics?

- *Convergence tests*
- *Change arithmetic*
- *Beware of the machine epsilon*
- *Epsilon could beat you before resolution is small enough to get a good result.*

- **Do diligence: you have some evidence of errors, it's your responsability to pound on it.**



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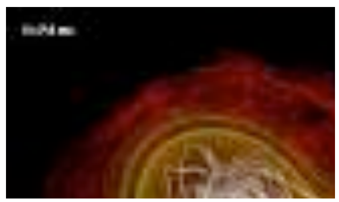
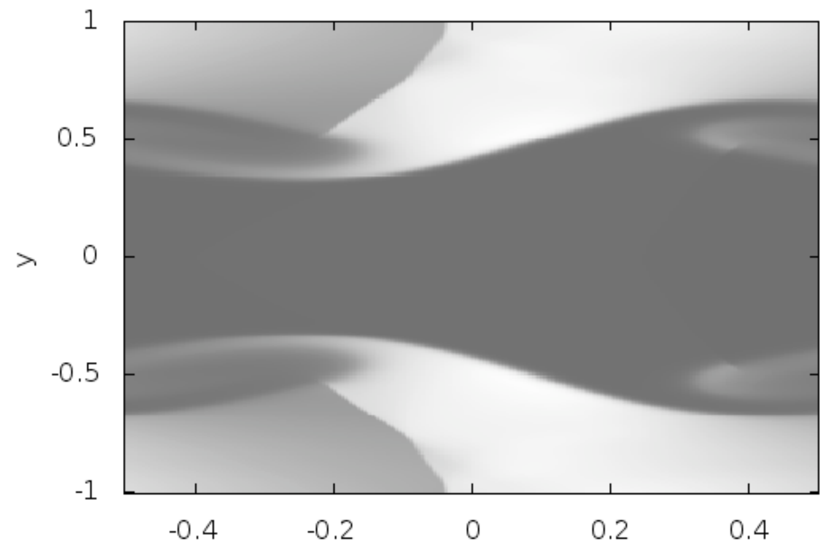
# Criticism to the state of the art

Formally, black holes will be f
been hear ringing….That means
predictions of GR at once.

About discovered black holes: w
other states of matter (strai



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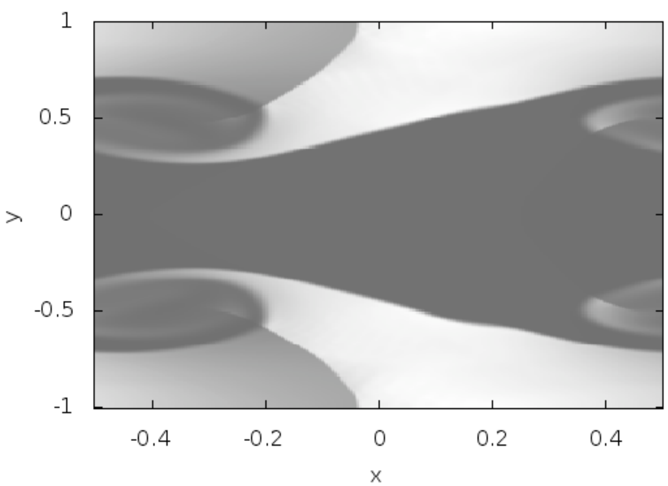
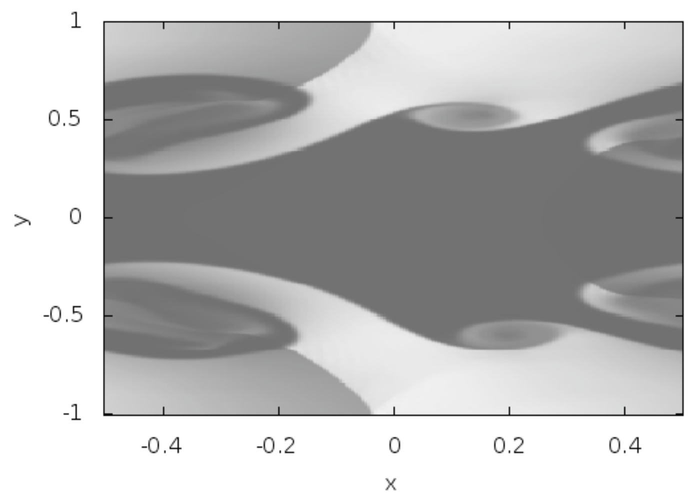


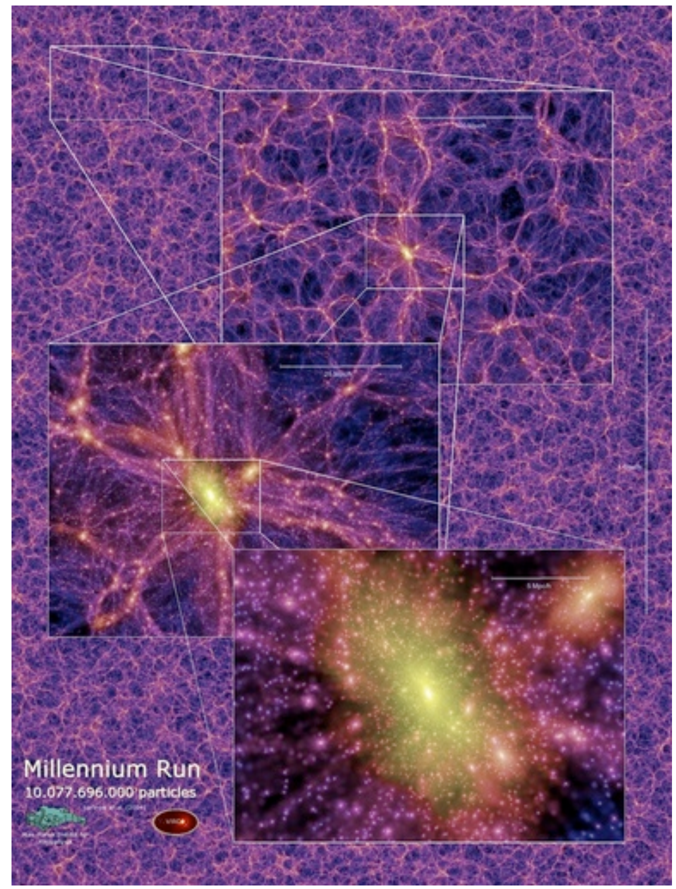

ch.mx

# Little samples

1411.xxxx: “the gas is expected to move with a speed of 10^6 Mach. With our code we model the gas with a velocity of !6 Mach”….This is about a problem of stellar-pulsar wind interaction.

1312.0598: influence of black holes is analyzed with a code that uses particles of 100solar masses …. In this case the most recent simulation is the “Lab”, there are No observations….

It is particularly annoying that No convergence tests are available for This problme (by the way we got the Expert at home: Juan Pablo Cruz Pérez)

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# Final comments

Once f[R] folks start working on the astrophsyics of their theories: BE SURE there will be a numerical relativist looking over their shoulders to tell them what is (or may be) wrong.

I do not see enough activity on trying to connect theories BEYOND GR with astrophysics. I'm sure there are a bunch of unemployed numerical relativists that may be willing to help. [it will depend on whether they are from the black-box school or from real schools that they may help].

Hopefully the scientific method is escaping from our hands with the irresponsible use of computer programs.

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THANKS

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# Status

Working on the MHD jets

Testing the GRHD mode

Plugging in the RMHD to curved fixed backgrounds (accretion of magnetized winds)

THANKS

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# The numerical problem ????????

GR has not been tested yet….

The idea is to use GR to model and understand astrophysical systems….

At the same time try to show it is correct…

About discovered black holes: we don't know whether there are other states of matter (stange stars)

Mientras tanto a darle a la astrofisica y relacionarla con observaciones…..

Densities: WD 10^9kg/m3, NS-core 10^17-10^18, BH 10^30, SUN-core 150,000

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